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Tests of Race Models for Reaction Time in Experiments with Asynchronous Redundant Signals

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Predictions of race models are derived for divided attention experiments in which redundant signals are presented at slightly different times. The models place constraints on the change in mean reaction time (RT) as a function of the time interval between signal onsets, and these constraints can be used to test race models within the redundant signals paradigm.

The redundant signals effect has often been used to test models of divided attention. This effect arises in target-detection tasks, where a subject must make a given response if presented with either one or both of two target signals, S_x and S_y (e.g., a tone and a light). Trials on which both signals are presented, S_{xy}, are called redundant signals trials, and the typical finding is that mean reaction time (RT) is smaller in these trials than in trials where either S_x or S_y is presented by itself. This "redundant signals advantage" was first reported by Todd (1912), who found that it decreases with increasing separation between the RT distributions produced by S_x and S_y in isolation. Subsequently, the redundant signals advantage was studied further as a means of quantifying intersensory facilitation (e.g., Hershenson, 1962).

Raab (1962) proposed that race models could explain the speedup of responses on redundant signals trials. Let RT_x, RT_y, and RT_{xy} be random variables representing the observed reaction times on trials in which the subject is presented with the signals S_x, S_y, and S_{xy}, respectively. According to race models, each signal initiates a separate process that will yield the response after some processing time, and the actual response is generated by the first of these processes to finish. Hence the model asserts

$$RT_{xy} = \min(RT_x, RT_y). \quad (1)$$

Such models are clearly compatible with the finding that

$$E[RT_{xy}] \leqslant \min(E[RT_x], E[RT_y]). \quad (2)$$

For example, Raab (1962) showed that this model predicts a redundant signals advantage of approximately 0.5 standard deviation units when RT_x and RT_y are independent and normally distributed. Because they provide a plausible and conceptually simple account of the redundant signals advantage, race models are a reasonable default model for this effect.

Miller (1978, 1982) noted that race models must predict

$$F_{xy}(t) \leqslant F_x(t) + F_y(t), \qquad t > 0, \quad (3)$$

where F_x, F_y, and F_{xy} are the cumulative distribution functions (CDFs) of RT_x, RT_y, and RT_{xy}, respectively. Luce (1986, p. 131) and Ulrich and Giray (1986) showed that this inequality still holds even if a residual base time is included in the analysis,[1] and Townsend and Nozawa (1995) have characterized more generally both the inequality and the models that do and do not satisfy it.

Many studies have used Inequality 3 to test race models (e.g., Diederich, 1992; Grice, Canham, & Boroughs, 1984; Miller, 1978, 1982, 1986; Mordkoff & Miller, 1993; Mordkoff & Yantis, 1991), because this inequality has provided the only general test of race models. Other tests have been constructed by assuming that RT_x and RT_y are independent or have specific distributions (e.g., Blake, Martens, Garrett, & Westendorf, 1980; Meijers & Eijkman, 1977; Raab, 1962), but these tests do not seem very satisfactory because of their

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[1] Let the detection time for S_x be T_x and for S_y be T_y. Furthermore, let the random variable M denote the motor time. Thus one may write $RT_x = T_x + M$, $RT_y = T_y + M$, and $RT_{xy} = \min(T_y, T_x) + M = \min(T_x + M, T_y + M) = \min(RT_x, RT_y)$. Therefore race models still predict (3) if a motor time is added on at the end of the race.

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extra assumptions. Experimental tests indicate that Inequality 3 is violated in a number of tasks, allowing race models to be ruled out for those tasks (e.g., Diederich, 1992; Grice *et al.*, 1984; Miller, 1978, 1982, 1986; Mordkoff & Miller, 1993; Mordkoff & Yantis, 1991). Such results are interpreted as evidence in favor of an alternative class of coactivation models, in which activation from both signals combines rather than races to produce the response. Coactivation models provide one class of alternatives to race models, and they seem particularly attractive for situations in which the data violate Inequality 3 (for further elaboration and specific examples of these models, see Diederich, 1992, 1995; Diederich & Colonius, 1991; Grice *et al.*, 1984; Schwarz, 1989, 1994; Townsend & Nozawa, 1995). However, there are other tasks in which Inequality 3 is satisfied, and in these tasks race models remain viable (e.g., Grice *et al.*, 1984; Grice & Reed, 1992; Meijers & Eijkman, 1977; Mordkoff & Yantis, 1991).

This article presents a new class of tests for race models that can sometimes reject them even when Inequality 3 is satisfied. In brief, we show that such models make predictions about how mean RT should change when a brief stimulus onset asynchrony (SOA) is introduced between signals on redundant signals trials. Previous studies have repeatedly demonstrated the value of SOA manipulations for illucidating elementary cognitive operations (e.g., Hershenson, 1962; Miller, 1986; Neely, 1977; Vorberg, 1985); the present article contributes to this trend by presenting a new way to use this manipulation for testing race models. This new test will not only strengthen the interpretation of SOA effects on RT but might also provide a convenient theoretical reference point for the development and comparison of alternative model classes.

As far as we know, the first mathematical analysis of race models for such a manipulation was provided by Heath (1984). He derived the prediction of mean RT as a function of SOA, when RT_x and RT_y are identically and exponentially distributed random variables. He showed that maximal RT facilitation occurs with synchronous signal onsets and that this facilitation diminishes monotonically as the onsets become more asynchronous.

In this paper we considerably extend this theoretical analysis. First, we show that race models imply under fairly general assumptions certain testable restrictions on the slope of the RT-SOA function. Second, we demonstrate with analysis and numerical examples that this new test supplements the CDF test (3) of race models. That is, the new test may allow race models to be rejected even though the CDF test does not, and vice versa. Thus, the two tests in combination provide a stricter diagnostic criteria for race models than does either test by itself. Third, we pursue a recent approach provided by Townsend and Nozawa (1995) to explore how capacity allocation in a processing system affects the properties of RT-SOA functions.

MEAN RT AS A FUNCTION OF SOA: PREDICTIONS OF RACE MODELS

Let S_x be presented at time t_x and S_y at $t_y = t_x + d$, where d represents the SOA. Note $d > 0$ means S_x is presented d msec before S_y, whereas $d < 0$ means S_y is presented d msec before S_x. Let $\mathrm{RT}(d)$ be an observed reaction time at a particular SOA, measured from the onset of the first signal, that is, from $t = \min(t_x, t_y)$.

The theoretical analysis in this section proceeds from a rather general version of the race model considered by Luce (1986) and Ulrich and Giray (1986; see also Townsend & Nozawa, 1995). According to this version, processing of S_x and S_y is accomplished in parallel detection channels up to a point in the processing system where each detection channel transmits its output into a single response channel. The detection times are T_x and T_y for S_x and S_y, respectively. Whichever detection channel finishes first activates the processing of the response channel. Thus, the race between detection channels finishes at the moment when the response channel becomes activated. Let M, the residual motor time, denote the processing duration of the response channel. According to this general version, then, the reaction time $\mathrm{RT}(d)$ for a given value of d is

$$\mathrm{RT}(d) = \begin{cases} \min(T_x, T_y + d) + M & \text{if } d \geqslant 0 \\ \min(T_x - d, T_y) + M & \text{if } d < 0 \end{cases} \tag{4}$$

where T_x, T_y, and M are random variables with arbitrary distributions.

The new test requires the assumption of SOA independence, which is formally stated in the following definition.

DEFINITION 1 (SOA Independence). Let $G_{x,y}(x, y \mid d) \equiv \Pr\{T_x \leqslant x \cap T_y \leqslant y \mid d\}$ be the joint cumulative distribution function of T_x and T_y for a given value of d. Furthermore, let $E[M \mid d]$ be the mean of motor time M for a given d. SOA independence is present when the following requirements (i) and (ii) are met for all values of d:

(i) $\mathrm{E}[M \mid d] = \mathrm{E}[M]$

(ii) $G_{x,y}(x, y \mid d) = G_{x,y}(x, y)$.

Remarks on Definition 1. Part (i) of Definition 1 requires that the mean of M does not vary with d. There is evidence that this requirement is satisfied: The lateralized readiness potential—a psychophysiological measure of motor activity derived from the electroencephalogram—develops over the same amount of time preceding responses to both single and redundant signals (Mordkoff, Miller, & Roch, 1996, Experiment 3). Note that neither the variance nor the higher moments of M are required to be independent of d. Thus, the requirement is not inconsistent with evidence that the variance of M varies slightly with d (Diederich & Colonius, 1987).

Part (ii) only requires that the joint distribution $G_{x,y}$ of T_x and T_y does not depend on d. Note that T_x and T_y can be dependent on each other and on M, and they can have different distributions.

Definition 1 is similar to the definitions of "context independence" and "perceptual separability," given by Colonius (1990) and Ashby and Townsend (1986), respectively, which are required by the CDF test (3) (Luce, 1986, pp. 128–131). Context independence is satisfied when the marginal distribution of RT_x (or RT_y) is identical for both S_x (or S_y) and S_{xy} trials. In contrast to Definition 1, this form of context independence is violated when the distribution of M varies with trial type. For instance, if the variance of M is different with S_x than with S_{xy}, the CDF test would be invalidated. For a thorough discussion of this form of context independence the reader is referred to Colonius (1990).

The following proposition forms the basis of this article. It shows that under race models the slope of the RT-SOA function is completely specified by the distribution of the difference $T_x - T_y$.

PROPOSITION 1 (The Slope of the RT-SOA Function). *Given the race model specified by Eq.* (4) *and the assumption that SOA independence* (*Definition* 1) *holds,*[2] *then the slope of* $\mathrm{E}[\mathrm{RT}(d)]$, *that is, its first derivative with respect to d,* $Z(d) = \partial \mathrm{E}[\mathrm{RT}(d)]/\partial d$, *depends on* G_{x-y}, *the CDF of the difference* $T_x - T_y$. *Specifically,*

$$Z(d) = \begin{cases} 1 - G_{x-y}(d) & \text{if } d \geqslant 0 \\ -G_{x-y}(d) & \text{if } d < 0. \end{cases} \tag{5}$$

Proof. We will only prove Proposition 1 for the case of $d \geqslant 0$; the proof for $d < 0$ is analogous. The expectation of $\mathrm{RT}(d)$ is given by

$$\mathrm{E}[\mathrm{RT}(d)] = \mathrm{E}[\min(T_x, T_y + d)] + \mathrm{E}[M] \tag{6}$$

and therefore the derivative with respect to d is

$$\frac{\partial \mathrm{E}[\mathrm{RT}(d)]}{\partial d} = \frac{\partial \mathrm{E}[\min(T_x, T_y + d)]}{\partial d}. \tag{7}$$

We will first derive a formula for $\mathrm{E}[\min(T_x, T_y + d)]$, and then obtain its derivative. The CDF of $\min(T_x, T_y + d)$ is

$$\Pr\{\min(T_x, T_y + d) \leqslant t\} = G_x(t) + G_y(t-d) - G_{x,y}(t, t-d) \tag{8}$$

[2] Strictly speaking, this proposition also requires that densities G_x, G_y, and $G_{x,y}$ do exist.

where $G_{x,y}$ denotes the bivariate CDF of T_x and T_y, G_x and G_y being the marginal CDFs of T_x and T_y. Because the minimum is a positive random variable, its expectation is

$$\mathrm{E}[\min(T_x, T_y + d)] = \int_0^\infty [1 - \Pr\{\min(T_x, T_y + d) \leqslant t\}]\, dt \tag{9}$$

$$= \int_0^\infty [1 - G_x(t) - G_y(t-d) + G_{x,y}(t, t-d)]\, dt. \tag{10}$$

(Because $G_y(t-d) = 0$ and $G_{x,y}(t, t-d) = 0$ for $t \leqslant d$, it is convenient to employ $t = 0$ as the lower integration limit in the line above and in the following lines.) Therefore, the derivative of $\mathrm{E}[\mathrm{RT}(d)]$ is

$$\frac{\partial \mathrm{E}[\mathrm{RT}(d)]}{\partial d} = \frac{\partial \int_0^\infty [1 - G_x(t) - G_y(t-d) + G_{x,y}(t, t-d)]\, dt}{\partial d} \tag{11}$$

$$= \int_0^\infty \left[-\frac{\partial G_y(t-d)}{\partial d} + \frac{\partial G_{x,y}(t, t-d)}{\partial d} \right] dt \tag{12}$$

$$= \int_0^\infty \left[g_y(t-d) + \frac{\partial \{\int_0^{t-d} \int_0^t g_{x,y}(x, y)\, dx\, dy\}}{\partial d} \right] dt \tag{13}$$

where $g_{x,y}$ is the joint density function of T_x and T_y, and g_y is the probability density function (PDF) of T_y.

Using the Leibniz rule for differentiating an integral, this reduces to

$$\frac{\partial \mathrm{E}[\mathrm{RT}(d)]}{\partial d} = 1 - \int_0^\infty \int_0^t g_{x,y}(x, t-d)\, dx\, dt \tag{14}$$

$$= 1 - \int_0^\infty \Pr\{T_x \leqslant t \cap T_y = t - d\}\, dt \tag{15}$$

$$= 1 - \int_0^\infty \Pr\{T_x \leqslant y + d \cap T_y = y\}\, dy \tag{16}$$

with a substitution of $y = t - d$. Then we may write

$$\Pr\{T_x \leqslant y + d \cap T_y = y\} = \Pr\{T_x \leqslant y + d \mid T_y = y\} \cdot g_y(y) \tag{17}$$

and

$$\frac{\partial \mathrm{E}[\mathrm{RT}(d)]}{\partial d} = 1 - \int_0^\infty \Pr\{T_x \leqslant y + d \mid T_y = y\}\ g_y(y)\ dy \tag{18}$$

$$= 1 - \Pr\{T_x \leqslant T_y + d\} \tag{19}$$

$$= 1 - \Pr\{T_x - T_y \leqslant d\} \tag{20}$$

$$= 1 - G_{x-y}(d). \tag{21}$$

This completes the proof.

Remarks on Proposition 1. Because $G_{x-y}(d)$ is non-decreasing in d for $d \geqslant 0$, it follows that the function $Z(d)$ decreases from $1 - G_{x-y}(0)$ to $1 - G_{x-y}(\infty) = 0$. Hence $\mathrm{E}[\mathrm{RT}(d)]$ must increase in a negatively accelerated fashion as d increases from 0 to ∞, toward the asymptotic value of $\mathrm{E}[T_x] + \mathrm{E}[M]$. When $d < 0$, $\mathrm{E}[\mathrm{RT}(d)]$ increases analogously as d decreases. Somewhat counterintuitively, this also implies that $\mathrm{E}[\mathrm{RT}(d)]$ has a V-shape, coming sharply to a local minimum at $d = 0$, which we will call the V-property.

Proposition 1 provides a means of testing race models, because it provides constraints on the slope of the function relating $\mathrm{E}[\mathrm{RT}(d)]$ to d. In particular, because $|Z(d)| \leqslant 1$, race models predict that a change in d of Δ msec should produce a change in mean RT of at most Δ msec, so any large change would justify rejection of such models.

Illustrative Examples of Proposition 1. We provide two numerical examples to illustrate the properties of Proposition 1. The first example assumes that the detection times T_x and T_y are independently and exponentially distributed with rates a and b, respectively. In this case the mean RT is

$$\mathrm{E}[\mathrm{RT}(d)] = \mathrm{E}[M] + \begin{cases} \dfrac{1}{a} + \left[\dfrac{1}{a+b} - \dfrac{1}{a}\right] e^{-ad} & \text{if } d \geqslant 0 \\[2ex] \dfrac{1}{b} + \left[\dfrac{1}{a+b} - \dfrac{1}{b}\right] e^{bd} & \text{if } d < 0 \end{cases} \tag{22}$$

with its first derivative (which is also identical to $G_{x-y}(d)$) equal to

$$Z(d) = \begin{cases} \dfrac{b}{a+b} e^{-ad} & \text{if } d \geqslant 0 \\[2ex] \dfrac{-a}{a+b} e^{bd} & \text{if } d < 0. \end{cases} \tag{23}$$

Panels A and B of Fig. 1 provide a numerical example for (22) and (23). First, as implied by Proposition 1, the mean RT increases in negatively accelerated fashion with d for $d > 0$ and with decreasing d for $d < 0$ illustrating the V-property. Second, as predicted by Proposition 1, the absolute value of slope $Z(d)$ of the RT-SOA function is always less than 1. The second example assumes independent and normally distributed detection times.[3] As expected on the basis of Proposition 1, an analogous result emerges for this case (see panels D and E).

Corollaries of Proposition 1. *Proposition* 1 *suggests three corollaries that might be useful in testing race models.*

Corollary 1. *If T_x and T_y are identically distributed, then $|Z(d)| \leqslant 0.5$.*

Proof. According to Proposition 1, $Z(0) = 1 - G_{x-y}(0)$. Now if T_x and T_y are identically distributed, $G_{x-y}(0) = 0.5$, and hence $Z(0) = 0.5$. In addition, G_{x-y} must increase as d increases, so $Z(d) \leqslant 0.5$ for $d \geqslant 0$. An analogous argument establishes the result $|Z(d)| \leqslant 0.5$ for $d < 0$.

Remark. In an experiment with equally detectable targets, this corollary provides a stricter test of race models than Proposition 1, because in this case $|Z(d)| \leqslant 0.5$. Thus, with equally detectable signals race models predict that a change in d of Δ msec should produce a change in mean RT of at most $\Delta/2$ msec.

Corollary 2. *For $d > 0$ define the average $U(d)$,*

$$U(d) = \frac{Z(d) + |Z(-d)|}{2}. \tag{24}$$

Given the assumptions of Proposition 1, *it follows that*

$$0 \leqslant U(d) \leqslant 0.5. \tag{25}$$

Proof. Inserting the results of Proposition 1 in Equation 24 gives

$$U(d) = \frac{1 - G_{x-y}(d) + G_{x-y}(-d)}{2}. \tag{26}$$

Since $G_{x-y}(d) \geqslant 0$ and $G_{x-y}(d) > G_{x-y}(-d)$ for all d, it follows immediately that $U(d) \leqslant 0.5$.

Remark. This corollary also provides a stricter test of race models than Proposition 1, because this test may even be violated when $|Z(d)| \leqslant 1$. Regardless of whether signals are equally detectable, the average RT obtained with signal offsets of d msec, $(\mathrm{E}[\mathrm{RT}(d)] + \mathrm{E}[\mathrm{RT}(-d)])/2$, should be at most $\mathrm{E}[\mathrm{RT}(0)] + d/2$ msec.

Examples (*continued*). Panels C and F of Fig. 1 show values of $U(d)$ computed by averaging values obtained at d

[3] Mean RT cannot be expressed explicitly when T_x and T_y are normally distributed. However, because the CDF of $T_x - T_y$ is known to be normal with mean $\mu_x - \mu_y$ and variance $\sigma_x^2 + \sigma_y^2$, Proposition 1 can be used to compute $\mathrm{E}[\mathrm{RT}(d)] = C + \int_{-\infty}^{d} Z(x)\,dx$, where the integration constant C must be equal to $\mathrm{E}[\mathrm{RT}(-\infty)]$.

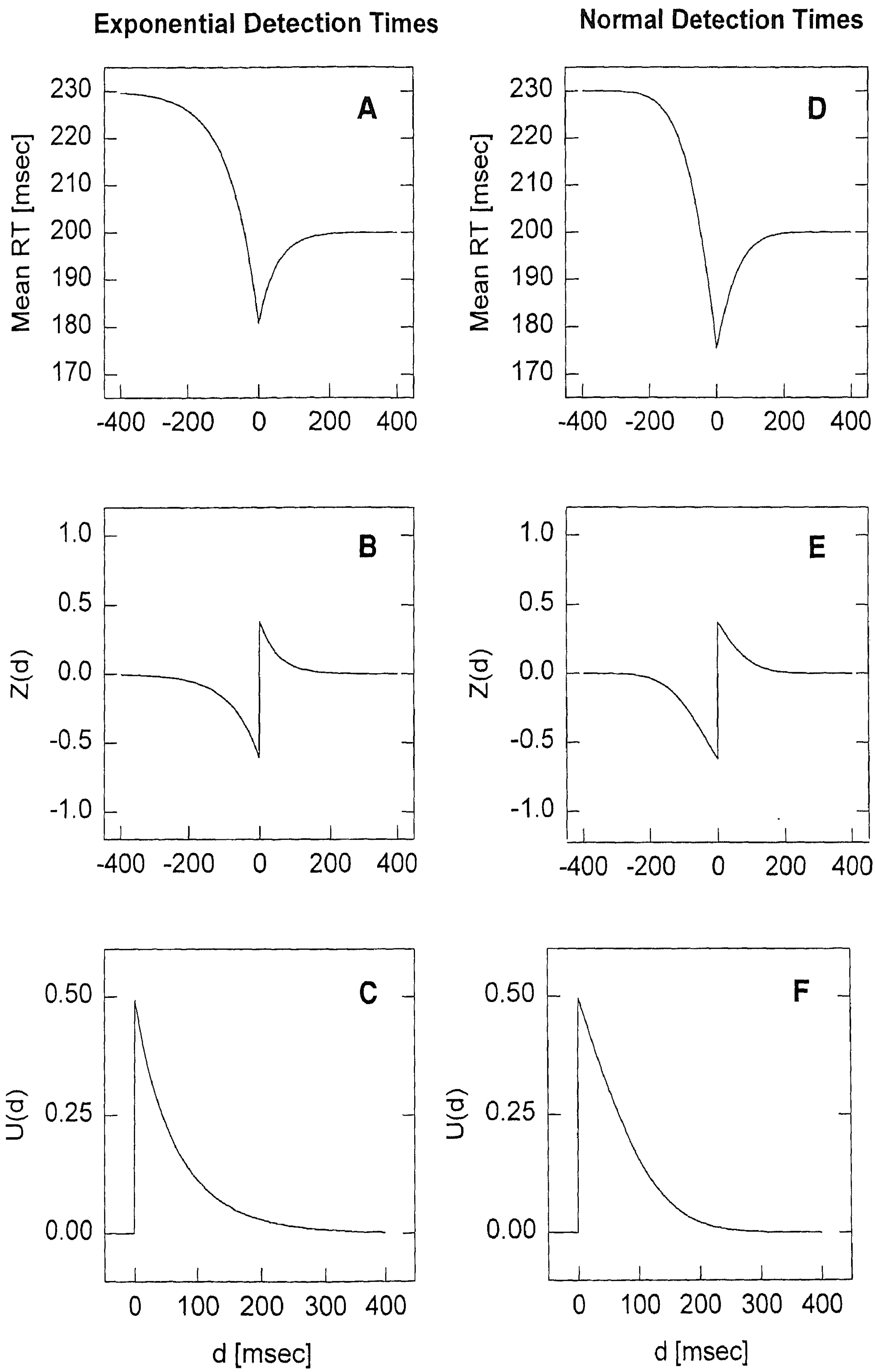

FIG. 1. Two numerical examples of race models. The examples show E[RT(d)] (Panels A and D), $Z(d)$ (Panels B and E), and $U(d)$ (Panels C and F) as a function of d. The left panels (A, B, and C) show the predictions of a race model in which the distributions of T_x and T_y were assumed to be exponential with parameters of $a = 1/50$ and $b = 1/80$ msec^{-1}, respectively, and E[M] is assumed to be 150 msec. The right panels (D, E, and F) show the predictions when T_x and T_y are normally distributed with parameters $\mu_x = 200$, $\sigma_x = 50$ and $\mu_y = 230$, $\sigma_y = 80$ *msec*.

and $-d$ for the corresponding lines in Panel B and E, respectively. It can be seen that $0 \leqslant U(d) \leqslant 0.5$ and that $U(d)$ decreases strictly from 0.5 to 0.0.

COROLLARY 3. $U(0) = 0.5$.

Proof.

$$U(0) = \frac{1 - G_{x-y}(0) + G_{x-y}(0)}{2} = 0.5. \qquad (27)$$

Remark. Given the assumptions of Proposition 1, then the function $U(d)$ must always begin with 0.5 at $d = 0$.

COROLLARY 4. *$U(d)$ is a decreasing function of d for $d > 0$.*

Proof.

$$U(d) = \frac{1 - G_{x-y}(d) + G_{x-y}(-d)}{2} \qquad (28)$$

$$= \frac{1 - [G_{x-y}(d) - G_{x-y}(-d)]}{2} \qquad (29)$$

$$= \frac{1 - \Pr\{-d < T_x - T_y < d\}}{2}. \qquad (30)$$

Note that $\Pr\{-d < T_x - T_y < d\}$ must increase with d. This completes the proof.

COMPARISON WITH THE RACE MODEL INEQUALITY

For practical purposes, it is of interest to see how the new SOA-based tests proposed here differ from the test based on CDFs (Inequality 3). Are there cases where one will reject race models and the other will not, or are they somehow logically (though not necessarily statistically) equivalent tests? Although we have not yet been able to characterize completely the relevant model space, we will present here three examples illustrating cases where one test would reject race models but the other would not. Together, these examples constitute a demonstration that, in this case, two tests really are better than one. (Table 1 provides a summary of these models.)

Triggered-Moment Models

Assume that the first signal that arrives at $t = 0$ triggers a moment, or time quantum, of duration Q (e.g., Baron, 1971; Efron, 1967; Sternberg & Knoll, 1973). A central processor registers any sensory input arriving during this moment, and it can only initiate a response when Q finishes. The speed of all succeeding processes increases with the amount of sensory input (and hence, the number of signals) arriving during this moment.

To apply this basic idea to the redundant signals effect, we assume that RT is the sum of Q and T_i, $i \in \{f, s\}$. The random variable T_i encompasses the duration of all processes after the central processor has initiated the response. Specifically, T_f is the duration when two signals arrive within the moment Q, whereas T_s is the duration when only one signal arrives during this moment.[4] Within this framework, it seems reasonable to assume that $E[T_s] - E[T_f] \geqslant 1$ msec.

For this model the mean RT for $d > 0$ is

$$\begin{aligned} E[RT(d)] &= E[Q + T_f \mid Q > d] \cdot \Pr\{Q > d\} \\ &\quad + E[Q + T_s \mid Q \leqslant d] \cdot \Pr\{Q \leqslant d\} && (31) \\ &= E[Q] + E[T_f] \cdot [1 - \Pr\{Q \leqslant d\}] \\ &\quad + E[T_s] \cdot \Pr\{Q \leqslant d\} && (32) \\ &= E[Q] + E[T_f] + E[T_s - T_f] \cdot F_Q(d), && (33) \end{aligned}$$

where $F_Q(d)$ denotes the CDF of Q. Therefore, the first derivative of $E[RT(d)]$ is

$$Z(d) = E[T_s - T_f] \cdot f_Q(d), \qquad (34)$$

where $f_Q(d)$ is the PDF of Q. (For $d < 0$ replace d by $-d$ in Eqs. (33) and (34)). Assuming that the variance of Q is relatively small, it is possible that $f_Q(d)$ is larger than one when d is equal to the mode of Q or at neighboring values near the mode. According to (34), $f_Q(d) > 1$ implies $Z(d) > 1$, so Proposition 1 would be violated in this case.

The preceding analysis of the triggered-moment model shows that the slope of the RT-SOA function is basically determined by the PDF f_Q of the time quantum Q. Interestingly, however, the model holds that violations of the CDF test are influenced by the CDFs G_s and G_f of T_s and T_f, respectively. In the following we will show this property.

First note that according to this model the RT to S_x is $RT_x = Q + T_s$, the RT to S_y is $RT_y = Q + T_s$, and the RT to S_{xy} is $RT_{xy} = Q + T_f$. Second, note that the CDF test is not violated if

$$F_{xy}(t) \leqslant F_x(t) + F_y(t) \qquad (35)$$

holds for all $t > 0$. If we assume that Q, T_f, and T_s are stochastically independent, then F_{xy} is the convolution of G_Q and G_f, that is, $F_{xy} = G_Q * G_f$. Likewise, F_x and F_y are the convolution of G_Q and G_s, that is, $F_x = G_Q * G_s$

[4] To simplify the math, we assume there are only two states f and s. In a more realistic yet less mathematically tractable version of this model, T_s and T_f could depend on the amount of accumulated sensory input during the period Q.

TABLE 1

Model's Predictions for the CDF and SOA Tests

Model	Brief Description	CDF Test	SOA Test
Race models	Faster of two racers determines *RT*	Always passed	Always passed
Triggered-moment models	Faster of two racers triggers a moment of duration *Q*. Sensory input arriving during this moment is accumulated. A response is initiated at the end of this moment, and response speed increases with the amount of accumulated sensory input.	May be violated	May be violated
Distinct-signals models	Similar to race models but additionally assumes a redundant racer in redundant-signals trials	May be violated	Always violated
Superposition model	Each stimulus starts a Poisson process. Detection occurs when a criterion of $c > 1$ pulses is reached.	Always violated	Always passed
Limited capacity models	Fixed amount of central capacity is shared between channels (exponential detection times)	Always passed	Always violated
Super capacity models	Double stimulation increases the amount of central capacity (exponential detection times)	Always violated	Always violated

and $F_y = G_Q * G_s$. Therefore, we may rewrite the former inequality as

$$G_Q * G_f(t) < 2 \cdot G_Q * G_s(t) \tag{36}$$

$$\int_0^t G_f(t') \cdot g_Q(t-t')\,dt' < 2 \cdot \int_0^t G_s(t') \cdot g_Q(t-t')\,dt' \tag{37}$$

$$\int_0^t [2 \cdot G_s(t') - G_f(t')] \cdot g_Q(t-t')\,dt' > 0. \tag{38}$$

It can be seen that the latter inequality will be satisfied if the relation

$$G_s(t) \geqslant 0.5 \cdot G_f(t) \tag{39}$$

holds between G_s and G_Q for all values of $t > 0$.

To summarize, according to the triggered-moment model, the slope of the RT-SOA function is determined by f_Q. Therefore, whether or not this model will violate the SOA test is determined by the shape of f_Q. If $f_Q(d) > 1$, then under regular conditions ($\mathrm{E}[T_s - T_f] > 1$) the SOA test will be violated. However, f_Q does not affect the outcome of the CDF test. This outcome is influenced by the distributions of T_s and T_f. Specifically, if (39) holds then the triggered-moment model will pass the CDF test (although it is not a race model by definition).

Figure 2 provides a numerical example of the triggered-moment model to illustrate these conclusions. For this example, we assumed that Q, T_f, and T_s follow normal distributions with means of 20, 230, and 250 msec and standard deviations of 5, 15, and 31 msec. For these parameter values, the model is consistent with Inequality 3 but violates Proposition 1 in two ways. First, the slope $Z(d)$ is equal to 1.6 at $d = 20$. (Note that the mean RT increases by almost 20 msec as the SOA increases from $d = 14$ to 26 msec.) Second, the model predicts a positively accelerated function relating mean RT to SOA, which is inconsistent with the V-property implied by Proposition 1.

Distinct-Signals Models

As a second illustration, we consider a model that is psychologically more plausible but mathematically less tractable. This model always violates the SOA test but violates the CDF test only under certain conditions. Specifically, this is a version of the distinct-signals model supported by the findings of Miller (1991). According to this model, three distinct channels may initiate a response. One corresponds to S_x, one corresponds to S_y, and one corresponds to S_{xy}, the combination of both signals. The racers of S_x and S_y start as soon as their respective signals come on. The redundant racer starts δ milliseconds after both signals are present, however, because the nervous system takes extra time to form a conjunction of the two signals. Let T_x, T_y, and T_r denote the processing times associated with each racer, then the distinct-signals model stipulates for $d > 0$

$$\mathrm{RT}(d) = \min(T_x, T_y + d, T_r + d + \delta) + M. \tag{40}$$

To obtain mathematically tractable expressions, it is convenient to assume that T_x, T_y, and T_r represent stochastically independent and exponentially distributed random variables with rates a, b, and c and CDFs F_x, F_y, and F_r. Furthermore, without loss of generality, we will ignore M. Under these assumptions the CDF of RT(d) for a given $d > 0$ is computed via

$$F_{xy}(t \mid d) = 1 - [1 - F_x(t)][1 - F_y(t-d)] \times [1 - F_r(t - d - \delta)] \tag{41}$$

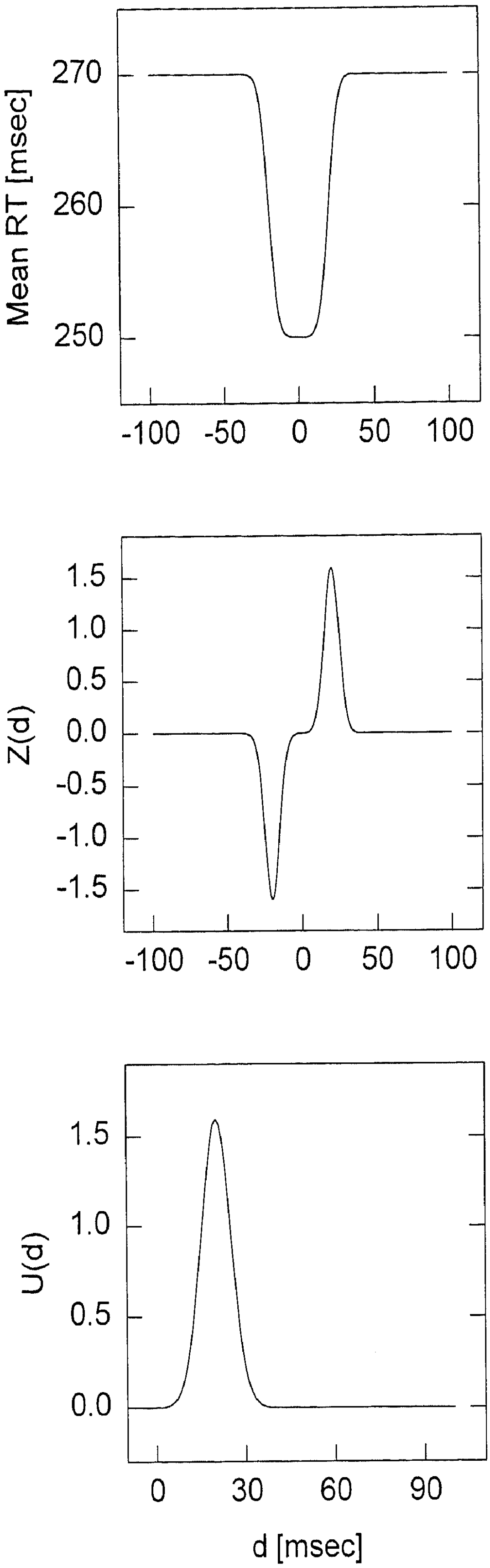

FIG. 2. A numerical example of the triggered-moment models. Panels A, B, and C show E[RT(d)], $Z(d)$, and $U(d)$ as functions of d. The variables Q, T_f, and T_x were assumed to be normally distributed with means 20, 230, and 250 msec and with standard deviations 5, 15, and 31 msec.

as

$$Fxy(t \mid d) = \begin{cases} 1 - e^{-at} & \text{if } t \leqslant d \\ 1 - e^{-at - b(t-d)} & \text{if } d < t \leqslant d + \delta \\ 1 - e^{-at - b(t-d) - c(t-d-\delta)} & \text{if } t > d + \delta. \end{cases} \tag{42}$$

The mean of RT(d) can be obtained from (42) by noting that

$$\text{E}[\text{RT}(d)] = \int_0^\infty [1 - F_{xy}(t \mid d)]\, dt \tag{43}$$

which yields, after integration and simplification,

$$\text{E}[\text{RT}(d)] = \frac{1 - e^{-ad}}{a} + \frac{1 - e^{-(a+b)\delta}}{a+b} e^{-ad} + \frac{e^{-(a+b)\delta - ad}}{a+b+c}. \tag{44}$$

For $d < 0$ the rates a and b have to be interchanged, and d must be replaced by $-d$. Figure 3 illustrates (44) and documents that RT(d) decreases when δ decreases or when c increases.

From (44) one obtains the slope of the RT-SOA function as

$$Z(d) = \frac{be^{-ad}(a+b+c) + ace^{-(a+b)\delta - ad}}{(a+b)(a+b+c)}. \tag{45}$$

From this result we finally arrive at the average absolute slope at $d = 0$

$$U(0) = \frac{1}{2} \cdot \left[1 + \frac{c}{a+b+c} e^{-(a+b)\delta} \right]. \tag{46}$$

It is easily seen from (46) that $U(0) > 0.5$ must hold for $c > 0$ and $\delta < \infty$. Therefore, the distinct-signals model will always violate the SOA test. It will not necessarily violate the CDF test, however, as we show next.

Note that the CDFs of RT_x and RT_y correspond to $F_x(t) = 1 - \exp[-at]$ and $F_y(t) = 1 - \exp[-bt]$, respectively. Furthermore, it follows from (42) that the CDF for RT_{xy} in a redundant signal trial with $d = 0$ is

$$F_{xy}(t \mid d = 0) = \begin{cases} 1 - e^{-(a+b)t} & \text{if } 0 < t \leqslant \delta \\ 1 - e^{-(a+b+c)t + c\delta} & \text{if } t > \delta. \end{cases} \tag{47}$$

The CDF test is passed if and only if $F_y(t) + F_x(t) \geqslant F_{xy}(t \mid d = 0)$ holds for all $t \geqslant 0$. First, consider the case $0 \leqslant t \leqslant \delta$, in which the CDF test can be written as

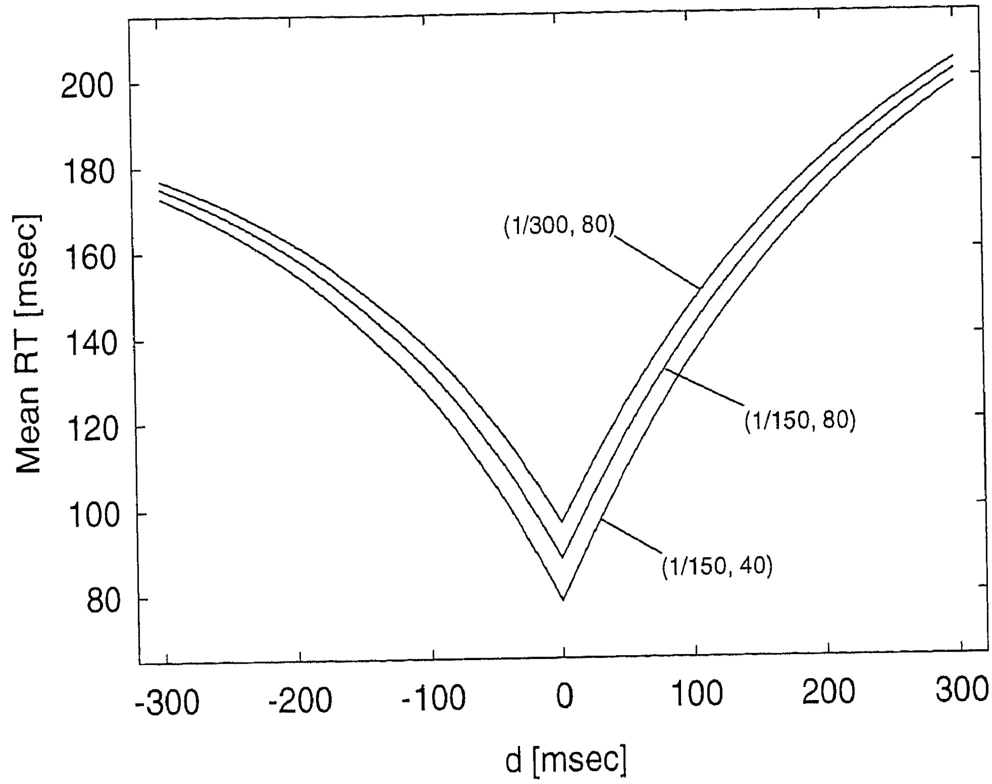

FIG. 3. A numerical example of the distinct-signals model. The figure shows E[RT(d)] as a function of d. All three curves were computed with rates a and b of 1/250 msec^{-1} and 1/200 msec^{-1}, respectively. The following parameter combinations (c, δ) were used for the various curves: (1/300, 80), (150, 80), and (150, 40).

$$1-e^{-at}+1-e^{-bt} \geqslant 1-e^{-(a+b)t} \tag{48}$$

$$1-e^{-at}-e^{-bt}+e^{-(a+b)t} \geqslant 0 \tag{49}$$

$$(1-e^{-at})\cdot(1-e^{-bt}) \geqslant 0 \tag{50}$$

and, therefore, will always be satisfied for all $t \in [0, \delta]$. Second, consider the case $t \geqslant \delta$, for which the CDF test will be satisfied if and only if the following inequality holds:

$$1-e^{-at}+1-e^{-bt} \geqslant 1-e^{-(a+b+c)t+c\delta}. \tag{51}$$

Unfortunately, (51) provides no explicit solution for the exact conditions under which the CDF test is satisfied. However, it is obvious that (51) can be violated for some values of $t \geqslant \delta$, when c is relatively large or when δ is relatively small.

This point is illustrated in Fig. 4. Each panel in this figure depicts the two functions $S(t) \equiv F_x(t)+F_y(t)$ and $F_{xy}(t \mid d=0)$. For all panels the rates a and b are 1/150 msec^{-1} and 1/200 msec^{-1}, respectively. In Panel A the values of c and δ were chosen such that (51) is satisfied, that is, $c=1/100$ msec^{-1} and $\delta=50$ msec. In Panel B the rate c was increased from 1/100 to 1/50 msec^{-1}, whereas in Panel C the delay δ was decreased from 50 to 25 msec. As can be seen in Panels B and C, both changes lead to a violation of the CDF test.

In sum, the distinct-signals model always violates the SOA test, but only violates the CDF test with certain combinations of parameters. The above analysis shows that the CDF test is violated when the processing speed in the redundant channel is relatively fast or when the nervous system establishes the conjunction relatively quickly. Hence, the SOA test is always able to distinguish between the distinct-signals model and standard race models defined by (4), but the CDF test is not.

Schwarz's (1989) *Superposition Model*

Schwarz's (1989) Poisson superposition model provides an example of a model in which Inequality 3 is violated but the new SOA-based test is not. According to this model, each stimulus starts a Poisson pulse generation process, and detection occurs when a criterion level of c pulses is reached.

Schwarz (1989, Eq. (5)) derived the mean of RT as a function of SOA for the superposition model, which is for $d>0$ given by

$$\mathrm{E}[\mathrm{RT}(d)] = \frac{c}{a} - \frac{b}{a(a+b)}\, e^{-ad} \times \sum_{i=0}^{c-1} \frac{(a\cdot d)^i}{i!}(c-i) + \mathrm{E}[M], \tag{52}$$

where rate a (b) denotes the rate of the Poisson process associated with stimulus S_x (S_y) and $c \in \{1, 2, 3, ...\}$ is the criterion count. (For $d<0$, d is replaced by $-d$, and the rates a and b are switched.)

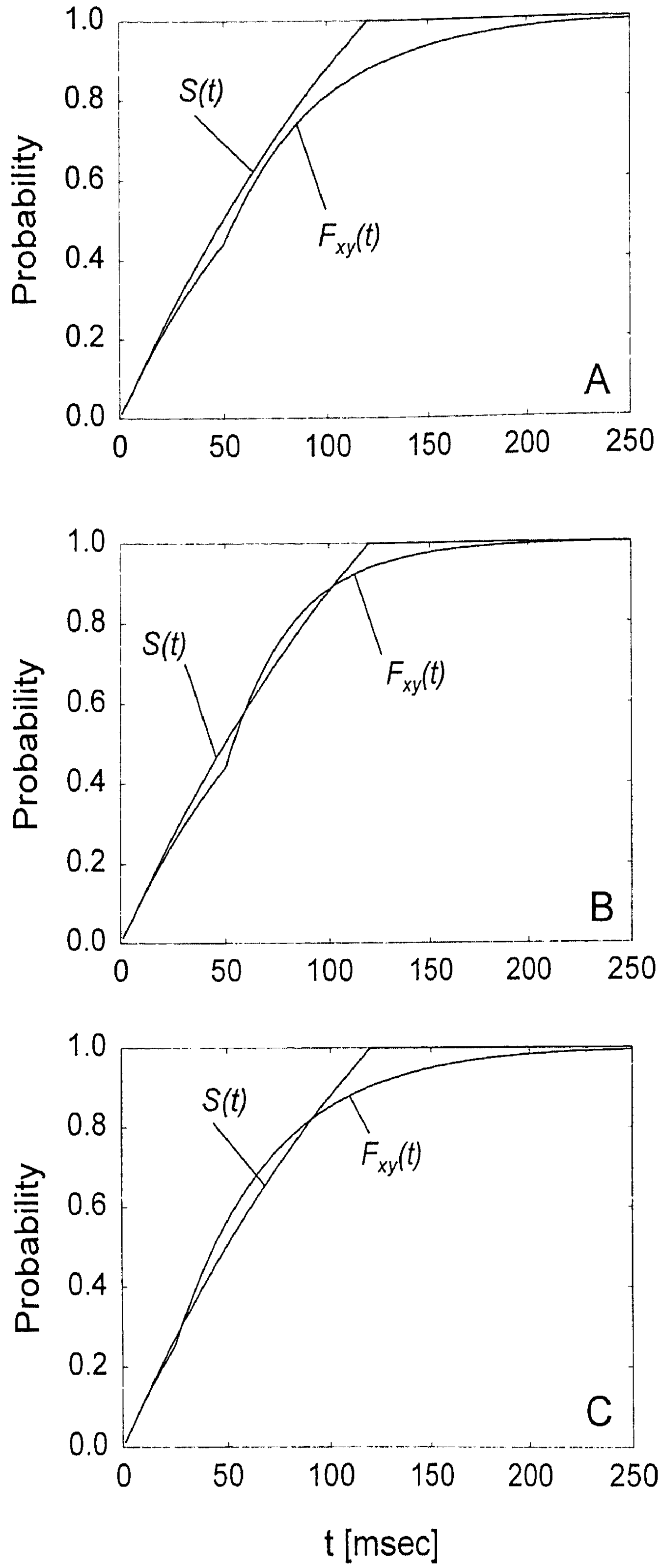

FIG. 4. A numerical example of the distinct-signals model. Each panel shows the CDF $F_{xy}(t \mid d = 0)$ and the sum $S(t) = F_x(t) + F_y(t)$. The curves in all three panels were computed with rates a and b of 1/150 msec^{-1} and 1/200 msec^{-1}, respectively. Different parameter combinations (c, δ) were used in the various panels. These combinations are (1/100, 50) for Panel A, (1/50, 50) for B, and (1/100, 25) for C. Note that the CDF test is violated in Panels B and C but not in A.

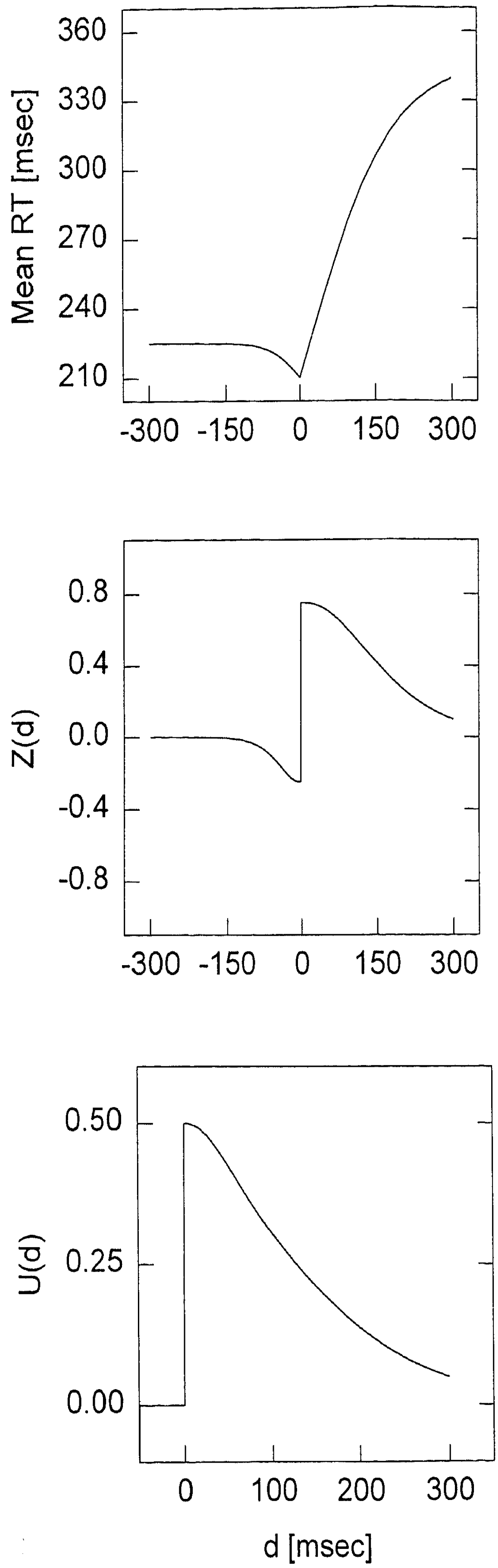

FIG. 5. A numerical example of Schwarz's (1989) superposition model, using best-fitting parameter estimates provided by Schwarz (1989, Table 1). Panels A, B, and C show E[RT(d)], $Z(d)$, and $U(d)$ as a function of d. Pulses are generated by signal S_x at rate $a_y = 1/20$ and by signal S_y at rate $a_x = 1/61$, so they are generated at rate 1/61 + 1/20 msec^{-1} on redundant-signals trials. These computations assume a criterion of $c = 3$ pulses needed to generate a response and a mean motor time of 165 msec.

The upper panel of Fig. 5 shows an example for (52). From this example it is clear that (a) the predicted mean RT as a function of d is within the bounds derived from race models, and (b) the model predicts a sharp local minimum mean RT at $d=0$, just as race models do. However, as shown by Townsend and Nozawa (1995, Proposition 7), such superposition models generally imply a violation of the CDF test. Thus, Schwarz's superposition model provides an instance of a non-race model which violates Inequality 3 but not the new SOA test.

Although the Poisson superposition model is mathematically rather intractable, additional analysis clearly suggests that it cannot violate the SOA test for any combination of the parameters a, b, and c. This conclusion is suggested by the predicted slope function, $Z(d)$, $d>0$,

$$Z(d)=\frac{b}{(a+b)\,a}\,e^{-ad} \times\left[c\cdot a+\sum_{i=1}^{c-1}\left[\frac{a^i d^{i-1}}{i!}\right](a\cdot d-i)(c-i)\right] \qquad (53)$$

which is obtained by differentiation of (52) and simplification. (For $d<0$, the remarks from above apply again.) The predicted slope function indicates that the steepest slopes are

$$\lim_{d\to 0+} Z(d)=\frac{b}{a+b}\leqslant 1.0 \qquad (54)$$

$$\lim_{d\to 0-} |Z(d)|=\frac{a}{a+b}\leqslant 1.0. \qquad (55)$$

In particular, this result shows that $U(0)=0.5$ must hold for all parameter combinations and is therefore consistent with the SOA test (although Schwarz's model is by definition a non-race model, except for the case $c=1$). In conclusion, then, the Poisson superposition model can mimic the SOA prediction of race models, although it can clearly be rejected by the CDF test as shown by Townsend and Nozawa (1995). The superposition model of Schwarz and its extension (Diederich, 1995) are instances of the independent channel summation model class defined by Townsend and Nozawa (1995). Future theoretical effort is necessary to see whether the conclusions obtained in this section for Schwarz's model can be generalized to the whole class of independent channel summation models.

CAPACITY ALLOCATION

Townsend and Nozawa (1995) provided a general mathematical framework to analyze the redundant-signals effect using the concept of system capacity, which is commonly invoked in cognitive models (e.g., Kahnemann, 1973; Norman & Bobrow, 1975; Navon & Gopher, 1979; Townsend & Ashby, 1978). In brief, their approach was to relate the amount of capacity available for processing redundant signals to that available for processing single signals, under the assumption that higher capacity leads to faster processing. As elaborated below, they showed that different assumptions about capacity lead to models that do or do not satisfy Inequality 3, like race or coactivation models. In this section we briefly review their approach; after that we utilize this framework to assess how capacity affects the slope of the RT-SOA function.

Townsend and Nozawa based the notion of capacity on the so-called integrated hazard functions $H_{xy}(t)$, $H_x(t)$, and $H_y(t)$, which are associated with the random variables $T_{xy}=\min(T_x, T_y)$, T_x, and T_y, respectively. Generally, the hazard function of a random variable with PDF $f(t)$ and CDF $F(t)$ is

$$h(t)=f(t)/[1-F(t)] \qquad (56)$$

and can be conceived of as its conditional density function. The hazard function often provides revealing information about the corresponding $f(t)$ of a random variable, although the two functions are mathematically equivalent, because $f(t)$ completely determines $h(t)$ and vice versa (cf. Luce, 1986, pp. 13–20; Townsend & Ashby, 1983, p. 26). The relation between both functions is

$$F(t)=1-e^{-H(t)}, \qquad (57)$$

where $H(t)$ is the integrated hazard function

$$H(t)=\int_0^t h(t')\,dt'. \qquad (58)$$

Function $H(t)$ can be conceptualized as the energy, that is, the work done by a processing channel within the interval $[0, t]$ (see Townsend & Ashby, 1983, Chap. 4; Townsend & Nozawa, 1995, p. 332).

As suggested by Townsend and Nozawa, independent race models provide a convenient reference point for developing the notion of capacity. As shown by the authors, race models imply the following simple expression $H_{xy}(t)=H_x(t)+H_y(t)$, which led the authors to the generalization

$$H_{xy}(t)=C(t)\cdot[H_x(t)+H_y(t)], \qquad (59)$$

where $C(t)$ is the *capacity coefficient* representing the processing efficiency in redundant-signals trials. Specifically, *unlimited capacity models* may be conceived as $C(t)=1$, *limited capacity* models as $C(t)<1$, and *super capacity* models as $C(t)>1$. In an unlimited capacity system the processing efficiency of one channel is not reduced if another channel begins its processing. In a limited capacity system, however, the efficiency of one channel would decrease if a second channel starts processing. In a super capacity system, the two channels may exert a mutual facilitation on each other such that processing efficiency in one channel actually increases when the second channel becomes active. For example, Townsend and Nozawa (1995) have shown that if a system is super capacity at all values of t, then the CDF test must be violated for some values of t. Thus, coactivation models (e.g., Schwarz, 1989) can be viewed as super capacity models within this framework.

Within the framework of capacity models, the question is, how does capacity affect the slope of the RT-SOA function? More specifically, which of the models (i.e., limited capacity, unlimited capacity, or super capacity) will violate the proposed SOA-test? The remainder of this section is devoted to an analysis of this issue.

Let $F_{xy}(t \mid d)$ be the CDF of the detection time $T_{xy}(d)$ in redundant-signals trials when signal S_x precedes signal S_y by d milliseconds. Furthermore, let $H_{xy}(t \mid d)$ be the integrated hazard function associated with $T_{xy}(t \mid d)$. Before we proceed with our analysis, it has to be clarified how $H_{xy}(t \mid d)$ can be related to $H_x(t)$ and $H_y(t)$ in the case of asynchronous signals. (We consider the analysis for $d > 0$ only. For $d < 0$, one just needs to reverse the subscripts x and y in the following analysis.) One obvious way to extend the above capacity definition to this case emanates from the hazard functions $h_x(t)$ and $h_y(t)$. Note that if S_y is delayed by d milliseconds relative to S_x, function $h_y(t)$ would merely be shifted by d milliseconds to the right along the time axis for independent race models, yielding $H_{xy}(t \mid d) = H_x(t) + H_y(t-d)$ with $H_y(t-d)$ with $H_y(t-d) = 0$ for $t < d$. Note that this expression can be rewritten as

$$H_{xy}(t \mid d) = H_x(d) + [H_x(t) - H_x(d) + H_y(t-d)], \qquad (60)$$

where the term in the brackets indicates the growth of $H_{xy}(t \mid d)$, when both signals are present. In accordance with Townsend and Nozawa, the preceding equation may be generalized to

$$H_{xy}(t \mid d) = \begin{cases} H_x(t) & \text{if } t \leqslant d \\ H_x(d) + C_d(t) \cdot [H_x(t) - H_x(d) + H_y(t-d)] & \\ \qquad \text{if } t > d \end{cases} \qquad (61)$$

for all values of $d > 0$. $H_x(d)$ represents the work done before the onset of the second signal. The function $C_d(t)$ measures the capacity at all values of time $t > d$ and has the same logical status as the capacity coefficient $C(t)$ provided by Townsend and Nozawa, although in the most general case the function $C_d(t)$ may be assumed to vary with d. It can also be seen that (61) implies these authors' definition of capacity for $d = 0$. As a matter of principle, the mean of $T_{xy}(d)$ could be computed via (61) and by noting that

$$E[T_{xy}(d)] = \int_0^\infty [1 - F_{xy}(t \mid d)]\, dt \qquad (62)$$

$$= \int_0^\infty e^{-H_{xy}(t \mid d)}\, dt. \qquad (63)$$

The generality of (61) hampers mathematical tractability. Therefore, we will employ two simplifying assumptions. First, we will assume a *fixed capacity* model, that is, $C(t) = c$ does not vary with time t (cf. Townsend & Ashby, 1978). Second, we assume that T_x and T_y follow an exponential distribution with rates a and b, respectively. With these additional assumptions the following theorem applies.

PROPOSITION 2. *Assume that* (i) *T_x and T_y are exponentially distributed with rates a and b, respectively, and* (ii) *$C_d(t) = c$ (fixed capacity assumption). Under both assumptions the mean of* $\mathrm{RT}(d) = M + T_{xy}(d)$ *is*

$$E[\mathrm{RT}(d)] = E[M] + \begin{cases} \dfrac{1}{a} + \left[\dfrac{1}{(a+b)\,c} - \dfrac{1}{a}\right] e^{-ad} & \text{if } d \geqslant 0 \\ \dfrac{1}{b} + \left[\dfrac{1}{(a+b)\,c} - \dfrac{1}{b}\right] e^{bd} & \text{if } d < 0. \end{cases} \qquad (64)$$

Proof. Proposition 2 will be proved for $d \geqslant 0$, as the proof for $d < 0$ is analogous. Since T_x and T_y are exponentially distributed, the integrated hazard functions are $H_x(t) = at$ and $H_y(t) = bt$, respectively. Therefore (61) can be written as

$$H_{xy}(t \mid d) = \begin{cases} at & \text{if } t \leqslant d \\ ad + c \cdot [at - ad + b(t-d)] & \text{if } t > d. \end{cases} \qquad (65)$$

With (63), the mean of T_{xy} is computed as

$$E[T_{xy}(d)] = \int_0^d e^{-at}\, dt + \int_d^\infty e^{-\{ad + c[at - ad + b(t-d)]\}}\, dt \qquad (66)$$

$$= \int_0^d e^{-at}\, dt + e^{-ad + c(a+b)\,d} \cdot \int_d^\infty e^{-c(a+b)}\, dt. \qquad (67)$$

Integration and simplifying yields the desired result. This completes the proof.

Figure 6 illustrates (64) for various values of c. It can be seen that the amount of RT facilitation increases with c. As one might expect for $c = 0.5$ (limited capacity), there is no facilitation in redundant-target trials.

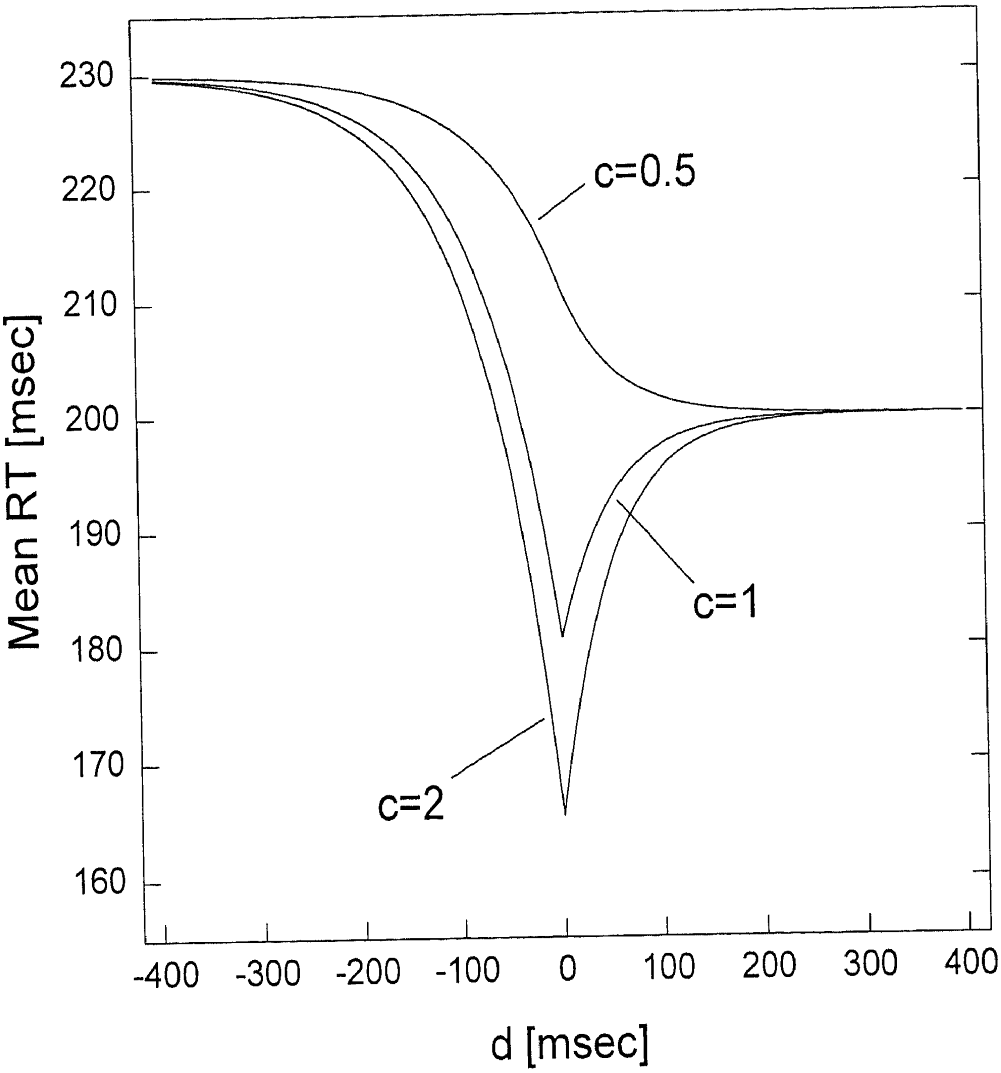

FIG. 6. Numerical examples for the capacity model. Mean RT as a function of SOA. Note that $c=0.5$ represents limited capacity, $c=1$ represents unlimited capacity, and $c=2.0$ represents super capacity. In all examples $a=1/50$, $b=1/80$ msec^{-1}, and $E[M]=150$ msec.

It seems instructive to consider the predicted slope of the RT-SOA functions $d=0$. Differentiation of (64) yields

$$Z(d)=\begin{cases}\left[1-\frac{a}{(a+b)\,c}\right]\cdot e^{-ad} & \text{if } d\geqslant 0\\ \left[\frac{b}{(a+b)\,d}-1\right]\cdot e^{bd} & \text{if } d<0\end{cases} \tag{68}$$

and therefore the slopes at $d=0$ of the RT-SOA function for synchronous signals is

$$\lim_{d\to 0+} Z(d)=1-\frac{a}{(a+b)\,c} \tag{69}$$

$$\lim_{d\to 0-} Z(d)=\frac{b}{(a+b)\,c}-1. \tag{70}$$

From this result the following corollary emerges.

Corollary 5. *Given the assumption stated in Proposition 2, the average absolute slope $U(d)$ at $d=0$ is*

$$U(0)=1-\frac{1}{2c}. \tag{71}$$

Proof. Proceeding from the definition of $U(d)$ given in Corollary 1, we can write

$$U(0)=\frac{\lim_{d\to 0+} Z(d)+|\lim_{d\to 0+} Z(d)|}{2} \tag{72}$$

$$=\frac{\left[1-\frac{a}{(a+b)\,c}\right]+\left[1-\frac{b}{(a+b)\,c}\right]}{2}. \tag{73}$$

Simplifying completes the proof.

From this corollary the following conclusion emerges: First, for the unlimited capacity models and in agreement with Corollary 2 the predicted average absolute slope at $d=0$ $U(0)=0.5$. Second, for limited capacity models we have $U(0)<0.5$ and for super capacity models $U(0)>0.5$. Thus the latter two classes imply violations of Corollary 2. These violations might be used to discriminate between model classes, at least under the present simplifying assumptions. Clearly, considerably more theoretical work on this topic is called for to assess the generality of this conclusion.

CONCLUSIONS

The purpose of this paper was to provide a new test for race models which focuses on the change of mean reaction time as the interval between two redundant targets changes. We have shown by example that this test can supplement the race model inequality; that is, the new test can rule out race models in situations where the race model inequality does not indicate a violation. On the other hand, the new test does not completely supercede the race model inequality, for there may be situations in which race models will be rejected via the inequality but not via the new test (e.g., Schwarz, 1989).

The main properties of the new test may be summarized as follows. First, the increase in mean RT as a function of SOA should never exceed a slope of one, i.e., $Z(d) \leqslant 1$. In an experiment with equally detectable targets, an even stricter bound applies, i.e., $Z(d) \leqslant 0.5$. Second, a stronger slope prediction is established if positive and negative SOA values are available. In this case, the average absolute slope $[Z(d)+|Z(-d)|]/2$ must be smaller than 0.5. Third, this average must always be 0.5 at $d=0$. Fourth, the slope for $d>0$ is strictly decreasing with increasing d and, analogously for $d<0$, strictly increasing as d decreases. This implies that the SOA function should have the shape of a V. In addition, this V-property requires that the minimum mean RT must always be located at $d=0$.

It is difficult to provide a general formal framework for the model class with which the SOA test is more sensitive than the CDF test. However, from the various non-race models that were analyzed in this article, the following tentative picture emerges:

(a) Both limited capacity and supercapacity models (Townsend & Nozawa, 1995) generally produce violations of the SOA test. Limited capacity models produce less increase in mean RT with increasing SOA than is predicted by standard race models, and supercapacity models predict more. Supercapacity models but not limited capacity models will also produce violations of the CDF test (see Townsend & Nozawa, 1995, p. 334), so the SOA test is particularly useful at detecting limited capacity processing.

(b) The triggered-moment and distinct-signals models suggest that the SOA test is also violated when the coactive effect of both channels needs some time to develop after both signals have been registered by the nervous system. In this case the relatively fast responses may appear to be produced in accordance with the race model. However, as time goes on a coactive affect or supercapacity builds up, so the relatively slow responses will be faster than the race model would predict. This buildup might be too late to have a strong effect on the left tail of F_{xy}, in which case the CDF test would not tend to be violated. The buildup would nevertheless profoundly affect the right tail of F_{xy}. Consequently, the mean RT in a redundant-signals trial would receive much more facilitation than predicted by the standard race model and hence the SOA test would be violated, despite the lack of violation of the CDF test. In any case, although it is difficult to provide a general framework of the models that violate the SOA but not the CDF test, it is clear that the SOA test provides an additional tool for discriminating not only non-race models from the standard race model but also between various classes of non-race models.

The new test is based on rather general assumptions. Like the race model inequality, the new test does not require that the detection times are uncorrelated random variables, nor does it require particular distributional assumptions about the detection times. In addition, the new test does not require distributional assumptions about the motor time. In contrast to the race model inequality, the diagnostic power of the new test is not affected by the variance contribution of the motor time (Ulrich & Giray, 1986).

The new test is based on the assumption of SOA independence which bears a strong similarity to the commonly made assumption of context independence for the CDF test (Colonius, 1990). SOA independence holds that neither $E[M]$ nor the joint distribution function of T_x and T_y depends on d. For example, both SOA and context independence would be violated if the mean of M were larger in single than in redundant-signals trials. Unfortunately, it seems extremely difficult to develop a test of race models without invoking this assumption. Nevertheless, even if the validity of this assumption is questioned, both the race model inequality and the new SOA test seem to be helpful tools in directing experimentation and data analysis to further enhance the interpretation of the redundant-signals effect.

It is difficult to apply the SOA test to previously reported RT data, because previous studies have used SOAs that were too large to provide a sensitive test of whether or not the average slope at the origin ($d=0$) differs from 0.5 (Diederich & Colonius, 1987; Giray & Ulrich, 1993; Miller, 1986; Schwarz, 1996). As far as we know, the only study with small changes in SOA is that of Diederich and Colonius (1987), who conducted a bimodal detection task with SOAs from 0 to 80 msec in steps of 10 msec. Unfortunately, only positive SOA values were used in this study. Nevertheless, the increase in mean RT when SOA was increased from 0 to

10 msec was almost 10 msec for all three subjects, suggesting a potential violation of the SOA test for the absolute average slope at $d = 0$. Future experimental studies should incorporate especially small SOA values (both positive and negative ones), particularly when the outcome of CDF test is negative, in order to use the SOA test to discriminate between race and non-race models.

REFERENCES

Ashby, F. G., & Townsend, J. T. (1986). Varieties of perceptual independence. *Psychological Review*, **93**, 154–179.

Baron, J. (1971). The threshold for successiveness. *Perception & Psychophysics*, **10**, 201–207.

Blake, R. R., Martens, W., Garrett, A., & Westendorf, D. (1980). Estimating probability summation for binocular reaction time data. *Perception & Psychophysics*, **27**, 375–378.

Colonius, H. (1990). Possibly dependent probability summation of reaction time. *Journal of Mathematical Psychology*, **34**, 253–275.

Diederich, A. (1992). *Intersensory facilitation*. Frankfurt am Main, Peter Lang.

Diederich, A. (1995). Intersensory facilitation of reaction time: Evaluation of counter and diffusion coactivation models. *Journal of Mathematical Psychology*, **39**, 197–215.

Diederich, A., & Colonius, H. (1987). Intersensory facilitation in the motor component? A reaction time analysis. *Psychological Research*, **49**, 23–29.

Diederich, A., & Colonius, H. (1991). A further test of the superposition model for the redundant signals effect in bimodal detection. *Perception & Psychophysics*, **50**, 83–86.

Efron, R. (1967). The duration of the present. *Annals of the New York Academy of Sciences*, **138**, 713–729.

Giray, M., & Ulrich, R. (1993). Motor coactivation revealed by response force in divided and focused attention. *Journal of Experimental Psychology: Human Perception and Performance*, **19**, 1278–1291.

Grice, G. R., Canham, L., & Boroughs, J. M. (1984). Combination rule for redundant information in reaction time tasks with divided attention. *Perception & Psychophysics*, **35**, 451–463.

Grice, G. R., & Reed, J. M. (1992). What makes targets redundant? *Perception & Psychophysics*, **51**, 437–442.

Heath, R. A. (1984). Response time and temporal oder judgement in vision. *Australian Journal of Psychology*, **36**, 21–34.

Hershenson, M. (1962). Reaction time as a measure of intersensory facilitation. *Journal of Experimental Psychology*, **63**, 289–293.

Kahneman, D. (1973). *Attention and effort*. Englewood Cliffs, NJ: Prentice-Hall.

Luce, R. D. (1986). *Response times: Their role in inferring elementary mental organization*. Oxford: Oxford University Press.

Meijers, L., & Eijkman, E. (1977). Distributions of simple RT with single and double stimuli. *Perception & Psychophysics*, **22**, 41–48.

Miller, J. O. (1978). Multidimensional same-different judgments: Evidence against independent comparisons of dimensions. *Journal of Experimental Psychology: Human Perception and Performance*, **4**, 411–422.

Miller, J. O. (1982). Divided attention: Evidence for coactivation with redundant signals. *Cognitive Psychology*, **14**, 247–279.

Miller, J. O. (1986). Timecourse of coactivation in bimodal divided attention. *Perception & Psychophysics*, **40**, 331–343.

Miller, J. O. (1991). Channel interaction and the redundant targets effect in bimodal divided attention. *Journal of Experimental Psychology: Human Perception and Performance*, **17**, 160–169.

Mordkoff, J. T., & Miller, J. O. (1993). Redundancy gains and coactivation with two different targets: The problem of target preferences and the effects of display frequency. *Perception & Psychophysics*, **53**, 527–535.

Mordkoff, J. T., Miller, J. O., & Roch, A. C. (1996). Absence of coactivation in the motor component: Evidence from psychophysiological measures of target detection. *Journal of Experimental Psychology: Human Perception and Performance*, **22**, 25–41.

Mordkoff, J. T., & Yantis, S. (1991). An interactive race model of divided attention. *Journal of Experimental Psychology: Human Perception and Performance*, **17**, 520–538.

Navon, D., & Gopher, D. (1979). On the economy of the human information processing system. *Psychological Review*, **86**, 214–255.

Neely, J. H. (1977). Semantic priming and retrieval from lexical memory: Roles of inhibitionless spreading activation and limited-capacity attention. *Journal of Experimental Psychology: General*, **106**, 226–254.

Norman, D. A., & Bobrow, D. G. (1975). On data-limited and resource-limited processes. *Cognitive Psychology*, **7**, 44–64.

Raab, D. H. (1962). Statistical facilitation of simple reaction times. *Transactions of the New York Academy of Sciences*, **24**, 574–590.

Schwarz, W. (1989). A new model to explain the redundant-signals effect. *Perception & Psychophysics*, **46**, 498–500.

Schwarz, W. (1994). Diffusion, superposition, and the redundant-targets effect. *Journal of Mathematical Psychology*, **38**, 504–520.

Schwarz, W. (1996). Further test of the interactive race model of divided attention: The effects of negative bias and varying stimulus—onset asynchronies. *Psychological Research*, **58**, 233–245.

Sternberg, S., & Knoll, R. L. (1973). The perception of temporal order: Fundamental issues and a general model. In S. Kornblum (Ed.), *Attention and Performance* (Vol. 4, pp. 629–685). New York: Academic Press.

Todd, J. (1912). Reaction to multiple stimuli. *Archives of Psychology*, **3**, 1–65.

Townsend, J. T., & Ashby, F. G. (1978). Methods of modeling capacity in simple processing systems. In J. Castellan and F. Restle (Eds.), *Cognitive theory* (Vol. III, pp. 200–239). Hillsdale, NJ: Erlbaum.

Townsend, J. T., & Ashby, F. G. (1983). *The stochastic modeling of elementary psychological processes*. Cambridge: Cambridge Univ. Press.

Townsend, J. T., & Nozawa, G. (1995). Spatio-temporal properties of elementary perception: An investigation of parallel, serial, and coactive theories. *Journal of Mathematical Psychology*, **39**, 321–359.

Ulrich, R., & Giray, M. (1986). Separate-activation models with variable base times: Testability and checking of cross-channel dependency. *Perception & Psychophysics*, **39**, 248–259.

Vorberg, D. (1985). Unerwartete Folgen von zufälliger Variabilität: Wettlauf-Modelle für den Stroop-Versuch. [Unexpected consequences of random variability: Competitive model for the Stroop phenomenon]. *Zeitschrift für Experimentelle und Angewandte Psychologie*, **32**, 494–521.

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