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Time resolution of clocks: Effects on reaction time measurement—Good news for bad clocks

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This paper investigates the measurement of reaction times (RTs) with clocks of limited time resolution. The questions raised are: (*a*) What is the relationship between measured and true RT? (*b*) Are mean and variance of measured RT biased, and if so, (*c*) how does this bias depend on the clock's time resolution? (*d*) Is it possible to correct this bias? It is concluded that the bias is practically negligible even if the time resolution of a clock is only 30 ms. The results show that a clock of limited time resolution biases mean and variance of measured RT. Furthermore it is shown that the effect of time resolution on detecting a true mean RT difference is negligible if the variance of true RT is relatively large. Formulae are provided to correct the bias of mean and variance of measured RT. In addition the implication of time resolution on measured RT for paired observations is analysed. It is shown that the product moment correlation coefficient but not the covariance of paired RT measures is affected by time resolution. A correction formula to remove the bias on the product moment correlation coefficient is provided.

1. Introduction

Consider a researcher reporting reaction time results from his study in a colloquium. The audience is fascinated by his new findings. At the end of the discussion following his talk someone asks for the time resolution of the clock applied for measuring the RTs. The speaker hesitates and finally confesses that his RT clock has only a time resolution of about 40 ms. Whispering of the audience is noticeable and one can feel that now everybody doubts his results. Is this doubt justified? The present paper provides an answer to this question.

Although this colloquium situation is rather hypothetical, we feel that many experimental researchers encounter this problem when they intend to measure RT with personal computers which provide built-in timing mechanisms with limited time resolution.

The purpose of this paper is threefold: (*a*) We examine the effect of time resolution on detecting mean RT differences and (*b*) the errors regarding mean and standard deviation of RT which may be caused by the limited time resolution of a clock.

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(c) We provide simple formulae to correct these errors. Before proceeding to these topics we define the measurement of RT with a counter clock and analyse the relationship between true and measured RT.

1.1. Measurement of RT with counter clocks

RT is a continuous random variable with an unknown density function† f_{RT}. However, if one measures RT with a counter clock the obtained measure is no longer continuous. To be concrete, when a stimulus requiring a response is presented the counter is reset to zero. Now the counter generates a discrete sequence of equally distant time points $\{t_1, t_2, t_3, \ldots\}$. The first time point t_1 occurs m ms after stimulus presentation. The interval between two consecutive time points defines the *time resolution m* of the counter clock; the larger m the worse the time resolution of the counter clock. Let t_i and t_{i+1} be the ith and $(i+1)$th count of the clock, then any event occurring during this interval is measured as time t_{i+1}. Therefore if the response occurs at time t with $t \in [t_i, t_{i+1})$ then response time t_{i+1} is recorded. RT* is always a multiple of time resolution m. Hence in the following considerations we distinguish between RT and RT*. It is intuitively clear that the true RT is better recoverable by RT* if m is small than if it is large. Furthermore the probability distribution functions (pdfs) F_{RT} and $F_{\mathrm{RT^*}}$ of RT and RT* respectively only coincide if $m=0$.

*1.2. The relationship between the distributions of RT and RT**

Recall that RT* is a discrete random variable with distinct values $(m, 2\cdot m, \ldots, i\cdot m, \ldots, v\cdot m)$ where v denotes the maximal count. v is selected in such a way that the probability $\Pr\{\mathrm{RT} \leqq v\cdot m\} = 1$ holds. The probability $\Pr\{\mathrm{RT^*} = i\cdot m\}$ of RT* can be obtained if m and the pdf F_{RT} of true RT is known

$$\begin{aligned}\Pr\{\mathrm{RT^*} = i\cdot m\} &= \Pr\{(i-1)\cdot m < \mathrm{RT} \leqq i\cdot m\} \\ &= F_{\mathrm{RT}}[i\cdot m] - F_{\mathrm{RT}}[(i-1)\cdot m] \qquad (1)\end{aligned}$$

for $i = 1, \ldots, v$.

Equation 1 can be used to compute the mean $E[\mathrm{RT^*}]$ and the variance $\mathrm{var}[\mathrm{RT^*}]$ of RT*

$$E[\mathrm{RT^*}] = \sum_{i=1}^{v} i\cdot m\cdot \Pr\{\mathrm{RT^*} = i\cdot m\} \qquad (2)$$

$$\mathrm{var}[\mathrm{RT^*}] = \sum_{i=1}^{v} (i\cdot m - E[\mathrm{RT^*}])^2 \cdot \Pr\{\mathrm{RT^*} = i\cdot m\}. \qquad (3)$$

†We will use throughout this paper the term *density function* to denote the first derivative of the probability distribution function F of a continuous random variable (cf. Kendall & Stuart, 1977, ch. I).

2. Effects of time resolution on RT results

RT studies are usually conducted to reveal the effects of one or more factors on mean RT. To simplify matters, assume one factor (e.g. signal intensity) with two factor levels l_1 (e.g. low intensity) and l_2 (e.g. high intensity). Let M_1 and M_2 denote the observed means of measured RT for factor levels l_1 and l_2 from independent samples respectively. Researchers usually perform a statistical test evaluating whether M_1 and M_2 differ significantly.

Proceed on the assumption that the hypothesis $\mathscr{H}_1$: $\mu_1 > \mu_2$ holds for true RTs. Then the power of a statistical test is defined as the probability of rejecting the null hypothesis $\mathscr{H}_0$ if the alternative, $\mathscr{H}_1$, actually holds. A test is said to be powerful if this probability is high.

Let us consider two researchers A and B; each conducts the above experiment. The only difference is that A uses an RT clock with a time resolution of 1 ms while B's clock has only a time resolution of 50 ms. If we proceed from realistic assumptions about true RTs (e.g. $\mu_1 = 220$, $\sigma_1 = 75$, $\mu_2 = 200$, $\sigma_2 = 70$ ms) then one might ask: Is B's test about as powerful as A's test or should B buy a better clock? The next section provides an answer.

2.1. *On detecting differences of mean true RTs*

Whether A or B rejects the hypothesis $\mathscr{H}_0$: $\mu_1 = \mu_2$ or, alternatively, accepts $\mathscr{H}_1$: $\mu_1 > \mu_2$, depends on the value of the statistics $U = M_1 - M_2$. If U exceeds a specified constant c_α then $\mathscr{H}_0$ is rejected. The constant c_α is specified such that the probability $\Pr\{U > c_\alpha | \mathscr{H}_0\} = \alpha$ is small, e.g. $\alpha = 0.025$, where $\Pr\{U > c_\alpha | \mathscr{H}_0\}$ denotes the probability of event $\{U > c_\alpha\}$ under the condition that $\mathscr{H}_0$ holds.

If M_1 and M_2 are based on a large number of observations n_1 and n_2 ($n_1, n_2 \geqq 30$) respectively then U is very nearly normally distributed according to the Central Limit Theorem even if RT* is not normally distributed. Furthermore, if $\mathscr{H}_0$ holds and σ_U denotes the standard deviation of U then $Z = (U - 0)/\sigma_U$ follows closely a standard normal distribution.† This information is needed to compute c_α

$$\Pr\{U > c_\alpha | \mathscr{H}_0\} = \Pr\left\{\frac{U-0}{\sigma_U} > \frac{c_\alpha - 0}{\sigma_U}\right\}$$

$$= \Pr\{Z > z_\alpha\}.$$

If this probability is equal to α, then it must be true that $c_\alpha = z_\alpha \cdot \sigma_U$. Therefore the rejection probability of $\mathscr{H}_0$, i.e. the power of the test, is given by

$$\Pr\{U > z_\alpha \cdot \sigma_U | \mathscr{H}_1\colon \mu_1 > \mu_2\}.$$

†Without loss of generality we assume that the value of σ_U is known to the experimenter. In a more realistic situation σ_U is of course unknown and therefore has to be substituted by the sample estimate $\hat{\sigma}_U$. In this case the random variable $Z = (U - 0)/\hat{\sigma}_U$ approaches only a standard normal distribution if the sample sizes are large. However, for our demonstrations the sampling distribution of Z has to be known. Therefore we will use σ_U instead of $\hat{\sigma}_U$.

For any given mean $\mu_U = E[M_1] - E[M_2]$, the random variable $Z' = (U - \mu_U)/\sigma_U$ has very nearly a standard normal distribution $\Phi(z) \equiv \Pr\{Z' \leqq z\}$. Hence if $\mathscr{H}_1$ holds then the rejection probability is

$$\Pr\{U > z_\alpha \cdot \sigma_U | \mathscr{H}_1\} = \Pr\left\{Z' > \frac{c_\alpha - \mu_U}{\sigma_U}\right\}$$
$$= 1 - \Phi\left[z_\alpha - \frac{\mu_U}{\sigma_U}\right] \tag{4}$$

with

$$\mu_U = E[M_1] - E[M_2] = E[\mathrm{RT}_1^*] - E[\mathrm{RT}_2^*]$$

and

$$\sigma_U^2 = \mathrm{var}[M_1 - M_2]$$
$$= \mathrm{var}[M_1] + \mathrm{var}[M_2] - 2 \cdot \mathrm{cov}[M_1, M_2], \tag{5}$$

where $\mathrm{cov}[M_1, M_2]$ denotes the covariance of M_1 and M_2. Since M_1 and M_2 are based on independent samples, we have $\mathrm{cov}[M_1, M_2] = 0$, and hence

$$\sigma_U^2 = \mathrm{var}[M_1] + \mathrm{var}[M_2]$$
$$= \frac{\mathrm{var}[\mathrm{RT}_1^*]}{n_1} + \frac{\mathrm{var}[\mathrm{RT}_2^*]}{n_2}. \tag{6}$$

Note that the quantities $E[\mathrm{RT}_1^*]$, $E[\mathrm{RT}_2^*]$, $\mathrm{var}[\mathrm{RT}_1^*]$ and $\mathrm{var}[\mathrm{RT}_2^*]$ are completely specified if time resolution m and the pdfs of RT_1 and RT_2 are known.

2.1.1. A numerical illustration. To evaluate the effect of m on the rejection probability $\Pr\{U > z_\alpha \cdot \sigma_U | \mathscr{H}_1\}$, the following steps were performed:

1. Special Erlangian distributions were used for RT_1 and RT_2 (cf. Townsend & Ashby, 1983). Empirical RT distributions look very similar to special Erlangian distributions. Therefore special Erlangian distributions have often been used to model RT distributions (see McGill, 1963). The pdf of a special Erlangian distribution is given by (cf. Cox, 1970, p. 20)

$$F_{\mathrm{RT}}(t) = 1 - e^{-\lambda \cdot t} \sum_{i=0}^{a-1} \frac{(\lambda \cdot t)^i}{i!}, \tag{7}$$

where a is a positive integer and $\lambda > 0$. The parameters a and λ specify the mean and variance of RT

$$E[\mathrm{RT}]=\frac{a}{\lambda}$$

$$\mathrm{var}\,[\mathrm{RT}]=\frac{a}{\lambda^2}.$$

2. Reasonable parameter values for a and λ were chosen for the pdfs of RT_1 and RT_2 respectively, such that mean and variance of RT_1 and RT_2 agreed with typical findings reported in the literature (cf. Burbeck & Luce, 1982; Krueger, 1984; Ulrich & Stapf, 1984). For ease of comparison two constraints were always used: (*a*) $E[\mathrm{RT}_1]-E[\mathrm{RT}_2]=20$ ms. (*b*) $n_1=n_2=n$; where n was adjusted in such a way that the rejection probability equals 0.90 for $m=0$ (perfect time resolution). This probability was selected arbitrarily for demonstration purposes and was also computed by equation (4).

3. A particular value of time resolution m was chosen.

4. The means $E[\mathrm{RT}_1^*]$ and $E[\mathrm{RT}_2^*]$ as well as the standard deviations $SD[\mathrm{RT}_1^*]$ and $SD[\mathrm{RT}_2^*]$ were computed via equations (1), (2), (3) and (7). The obtained values were used in equation (4) to compute the rejection probability.

5. Steps 3 and 4 were carried out for $m=2$, 4, 8, 16, 32, 64 and 128 ms.

Results. The results of these computations are shown in Fig. 1 for three different pairs of pdfs F_{RT_1} and F_{RT_2}. These pairs differ regarding to the variance of RT: small, medium and large RT variance. Somewhat surprising is the finding that the rejection probability for these examples is almost unaffected by time resolution if $m<64$ ms. Whether the rejection probability declines fast or slowly toward zero with m depends, all other things being equal,† on RT variance: the decline is faster for small RT variance. The SDs used for our computations can be regarded as small, relative to those reported in typical RT studies (cf. Ulrich & Stapf, 1984; but see also Kristofferson, 1976). Therefore, our computations demonstrate that a clock with a time resolution of only 30 ms may be sufficient in most cases.

2.2. *Effects of time resolution on mean and variance*

Does $E[\mathrm{RT}^*]$ agree with $E[\mathrm{RT}]$, and $SD[\mathrm{RT}^*]$ with $SD[\mathrm{RT}]$ or does time resolution m produce systematic biases on $E[\mathrm{RT}^*]$ and $SD[\mathrm{RT}^*]$? Table 1 provides an answer to this question:

A special Erlangian distribution with $E[\mathrm{RT}]=80$ and $SD[\mathrm{RT}]=21$ ms was assumed for true RT. Equations (2) and (3) were used to compute $E[\mathrm{RT}^*]$ and $SD[\mathrm{RT}^*]$ respectively for different values of m. As one can see in Table 1, $E[\mathrm{RT}^*]$ and $SD[\mathrm{RT}^*]$ increased with m; a smaller effect of m is observed for $SD[\mathrm{RT}^*]$ than for $E[\mathrm{RT}^*]$.

†Note that the sample sizes n_1 and n_2 differ between the curves in Fig. 1 to equalize the rejection probabilities at $m=0$ for all curves.

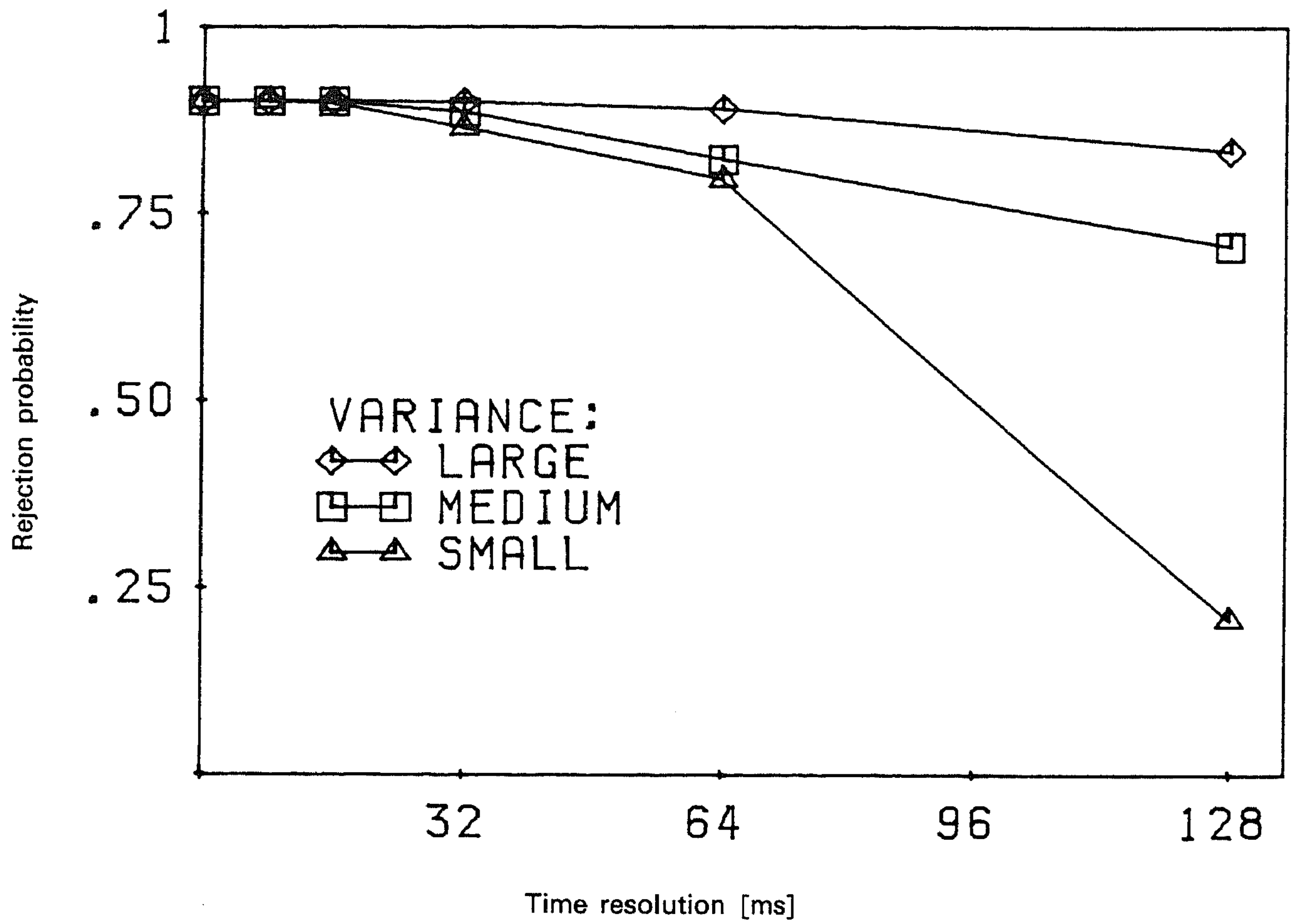

Figure 1. Rejection probability $\Pr\{U > c_{0.025} | \mathscr{H}_1\}$ as a function of time resolution for three pairs of distributions. Upper graph (large variance): $E[\mathrm{RT}_1]=220$, $E[\mathrm{RT}_2]=200$, $SD[\mathrm{RT}_1]=110$, $SD[\mathrm{RT}_2]=82$ ms, and $n_1=n_2=494$. Middle graph (medium variance): $E[\mathrm{RT}_1]=120$, $E[\mathrm{RT}_2]=100$, $SD[\mathrm{RT}_1]=38$, $SD[\mathrm{RT}_2]=33$ ms, and $n_1=n_2=68$. Lower graph (small variance): $E[\mathrm{RT}_1]=80$, $E[\mathrm{RT}_2]=60$, $SD[\mathrm{RT}_1]=27$, $SD[\mathrm{RT}_2]=20$ ms, and $n_1=n_2=30$.

Table 1. Effects of time resolution m on $E[\mathrm{RT}^*]$ and $SD[\mathrm{RT}^*]$

m	$E[\mathrm{RT}^*]$	Corrected	$SD[\mathrm{RT}^*]$	Corrected
0	80	–	21	–
2	81	80	21	21
4	82	80	21	21
8	84	80	22	21
16	88	80	22	21
32	96	80	23	21
64	114	82	30	23

2.2.1. Bias correction: Adaptation of Sheppard's method. Fortunately the bias produced by time resolution m can be corrected quite satisfactorily. A slight modification of Sheppard's method for grouping (see Kendall & Stuart, 1977, pp. 77–82) can be used to correct the measured means and variances. The rationale behind this method has to be elucidated:

Moments of a continuous random variable are often estimated on the basis of a frequency distribution with individual values classified into equal sized intervals, that is, into groups. Each group frequency is assumed to be concentrated at the mid-point of the corresponding interval. When the estimation of moments is based on group frequencies and corresponding mid-points, a certain amount of bias is introduced for all moments of second and higher order. This bias increases with the size of the intervals.

Sheppard's method can be used to correct this bias. Because the distribution of RT* can be regarded as a grouped frequency distribution with interval size m, an adaptation of Sheppard's method for grouping can be used to recover the mean and SD of true RT from RT*.

To adapt this method, the first step is to introduce the random variable RT^c, where the superscript c denotes the measurement of a hypothetical clock, which indicates the mid-point $(t_i+t_{i+1})/2$ if true RT falls into the ith interval $[t_i, t_{i+1})$. Sheppard's method is directly applicable to this hypothetical measurement to uncover mean and variance of true RT (see Kendall & Stuart, 1977, p. 78)†

$$E[\mathrm{RT}]=E[\mathrm{RT}^c] \tag{8}$$

$$\mathrm{var}\,[\mathrm{RT}]=\mathrm{var}\,[\mathrm{RT}^c]-\frac{m^2}{12}. \tag{9}$$

In the second step we note that the relationship

$$\mathrm{RT}^c=\mathrm{RT}^*-\frac{m}{2} \tag{10}$$

must hold between RT* and RT^c. Substituting equation (10) into equation (8) and (9) yields the desired result to uncover mean and variance of RT from RT*

$$E[\mathrm{RT}]=E\left[\mathrm{RT}^*-\frac{m}{2}\right]$$

$$E[\mathrm{RT}]=E[\mathrm{RT}^*]-\frac{m}{2} \tag{11}$$

and since translation does not affect variance

†Of course, Sheppard's method can also be used to uncover third and higher central moments of true RT. The interested reader should refer to the formulae provided by Kendall & Stuart (1977, p. 78).

$$\mathrm{var}\,[\mathrm{RT}] = \mathrm{var}\left[\mathrm{RT}^* - \frac{m}{2}\right] - \frac{m^2}{12}$$

$$= \mathrm{var}\,[\mathrm{RT}^*] - \frac{m^2}{12}. \qquad (12)$$

Equation (12) can also be used to explain the finding above that the power of the test to detect mean differences (i.e. rejection probability) is more attenuated by time resolution m for smaller than for larger RT variances. From equation (12) it follows that

$$\mathrm{var}\,[\mathrm{RT}^*] = \mathrm{var}\,[\mathrm{RT}] + \frac{m^2}{12}. \qquad (13)$$

From this one can see that the contribution of the term $m^2/12$ to var [RT*] is negligible if var [RT] is large. Therefore σ_U [see equation (6)] will depend less on time resolution m if var [RT] is large and hence the rejection probability [see equation (4)] is almost unaffected by m.

One might also wonder why a counter clock with a time resolution of 30 ms or more is sufficient for comparing reaction times that differ by 20 ms as in our computations above? If RT is a continuous random variable with a density function that covers several clock intervals then RT* has a discrete distribution with mean $E[\mathrm{RT}^*] = E[\mathrm{RT}] + m/2$ according to equation (11). Therefore the difference between the means of RT_1^* and RT_2^* is equal to the difference of the corresponding true means, that is, the measured difference does not depend on m.

2.2.2. The relative bias of SD[RT*]. We used equations (11) and (12) for several theoretical distributions and found that the bias on means and variances can be corrected very satisfactorily (see Table 1). However, it should be noted that the quality of correction was found to decrease with increasing m. In all our computations it turned out that the correction formulae worked very satisfactorily if *SD*[RT] is less or equal to time resolution m.

In general the bias of *SD*[RT] will be less if the variance of RT is large compared to time resolution m. To show this, assume that $SD[\mathrm{RT}] = k \cdot m$. Then one derives from equation (12)

$$\frac{SD[\mathrm{RT}^*] - SD[\mathrm{RT}]}{SD[\mathrm{RT}]} = \sqrt{1 + \frac{1}{12k^2}} - 1 \qquad (14)$$

expressing the difference $SD[\mathrm{RT}^*] - SD[\mathrm{RT}]$ relative to *SD*[RT] as a function of k. For k equal to 1, 2, 3, 4, 5 and 6 one obtains the values 0.041, 0.01, 0.005, 0.003, 0.002 and 0.001 of $(SD[\mathrm{RT}^*] - SD[\mathrm{RT}])/SD[\mathrm{RT}]$ respectively. It can be seen from this computation that *SD*[RT*] and *SD*[RT] agree satisfactorily as long as $SD[\mathrm{RT}] \geqq m$.

For example, if it is assumed that $SD[\mathrm{RT}]=60\,\mathrm{ms}$ and the time resolution of an available clock is $m=30\,\mathrm{ms}$ then k is computed by $k=SD[\mathrm{RT}]/m=60/30=2$. Inserting $k=2$ into equation (14) yields

$$\frac{SD[\mathrm{RT}^*]-SD[\mathrm{RT}]}{SD[\mathrm{RT}]}=0.01,$$

showing that the relative difference between true and measured SD is negligible.

2.2.3. The higher-order contact requirement. It should be stressed that Sheppard's method is only applicable if the pdf of RT satisfies the so-called *higher-order contact* requirement (see Kendall & Stuart, 1977, pp. 78–80). According to this requirement the pdf of RT must contact the abscissa at the terminals of the range within which reaction time is observed. For example an exponential distributed random variable (cf. Luce, 1986, p. 10) does not satisfy this requirement because it has a higher-order contact at one end but not at the start of the curve, being in fact J-shaped and very abrupt at this point. Since reaction time distributions are usually neither J- nor U-shaped this requirement should be met in almost every case.

2.3. Additional notes on dependent measures

In the computations of rejection probabilities it was assumed that the means M_1 and M_2 were based on independent observations. Therefore one might ask whether the obtained results can be generalized to dependent observations. This section provides an answer.

Suppose that the pair $(\mathrm{RT}^*_{1i}, \mathrm{RT}^*_{2i})$ represents the measured RTs of the ith subject $(i=1,\ldots,n)$ under two conditions l_1 and l_2, respectively, and let be $U_i=\mathrm{RT}^*_{1i}-\mathrm{RT}^*_{2i}$. Further assume that the sequence $(\mathrm{RT}^*_{11}, \mathrm{RT}^*_{21}),\ldots,(\mathrm{RT}^*_{1n}, \mathrm{RT}^*_{2n})$ represents a random sample from some joint distribution $\Pr\{\mathrm{RT}^*_1 \leqq t_1 \cap \mathrm{RT}^*_2 \leqq t_2\}$ of RT^*_1 and RT^*_2 (cf. Kendall & Stuart, 1977, pp. 20–22). Since the n differences $U_1,\ldots,U_n$ pertain to different subjects, these differences will be independent random variables. Again, we wish to test $\mathscr{H}_0$: $E[U]=0$.

In this case the rejection probability is also computed via equation (4), however, with a slight modification since M_1 and M_2 are correlated. Therefore for the calculation of σ_U according to equation (5) the term $\mathrm{cov}[M_1, M_2]$ must be taken into account. It is obvious that the above results related to the rejection probability $\Pr\{U>z_\alpha\cdot\sigma_U|\mathscr{H}_1\}$ can only be generalized to dependent measures if $\mathrm{cov}[M_1, M_2]$ does not vary with time resolution m.

Theorem 1 (Invariance Property). If M_1 and M_2 are measured with a clock of time resolution m and given the higher-order contact requirement for the pdfs of RT_1 and RT_2 holds, then $\mathrm{cov}[M_1, M_2]$ does not depend on m but only on sample size n and on the covariance of RT_1 and RT_2, that is,

$$\mathrm{cov}[M_1, M_2] = \frac{\mathrm{cov}[\mathrm{RT}_1, \mathrm{RT}_2]}{n}. \tag{15}$$

Proof

$$\mathrm{cov}[M_1, M_2] = \mathrm{cov}\left[\frac{\sum_{i=1}^n \mathrm{RT}^*_{1i}}{n}, \frac{\sum_{j=1}^n \mathrm{RT}^*_{2j}}{n}\right].$$

This expression can be rewritten as (cf. Mood, Graybill & Boes, 1974, p. 179, Theorem 2)

$$\mathrm{cov}[M_1, M_2] = \frac{1}{n^2}\sum_{i=1}^n \sum_{j=1}^n \mathrm{cov}[\mathrm{RT}^*_{1i}, \mathrm{RT}^*_{2j}].$$

If $i \neq j$ then RT_{1i} and RT_{2j} are independent random variables. Hence

$$\mathrm{cov}[M_1, M_2] = \frac{1}{n}\mathrm{cov}[\mathrm{RT}^*_1, \mathrm{RT}^*_2].$$

Using equation (10)

$$\begin{aligned}\mathrm{cov}[M_1, M_2] &= \frac{1}{n}\mathrm{cov}\left[\mathrm{RT}^c_1 + \frac{m}{2}, \mathrm{RT}^c_2 + \frac{m}{2}\right]\\ &= \frac{1}{n}\mathrm{cov}[\mathrm{RT}^c_1, \mathrm{RT}^c_2].\end{aligned}$$

Wold (1934) generalized Sheppard's method to bivariate moments (see Kendall & Stuart, 1977, pp. 86–87). His generalization is also directly applicable to uncover the bivariate moments of RT_1 and RT_2 from the bivariate moments of RT^c_1 and RT^c_2. Wold (1934) showed that

$$\mathrm{cov}[\mathrm{RT}_1, \mathrm{RT}_2] = \mathrm{cov}[\mathrm{RT}^c_1, \mathrm{RT}^c_2]$$

which completes the proof of Theorem 1.

Theorem 1 shows that $\mathrm{cov}[M_1, M_2]$ is not changed by the clock's time resolution m. The rejection probability given by equation (4) depends on $\mathrm{cov}[M_1, M_2]$ which is unaffected by m according to the Invariance Property. As in the case of independent measures the rejection probability is only attenuated because the variances of RT^*_1 and RT^*_2 increase with m.

2.3.1. Remarks on the correlation coefficient of RT^*_1 *and* RT^*_2. Although the covariance of RT^*_1 and RT^*_2 does not depend on time resolution m, the Pearson product moment correlation coefficient ρ^* of RT_1^* and RT_2^*, however, does. In general, ρ^* is attenuated as m increases, which can be easily shown

$$\rho^* = \frac{\mathrm{cov}[RT_1^*, RT_2^*]}{SD[RT_1^*] \cdot SD[RT_2^*]}$$

$$= \frac{\mathrm{cov}[RT_1, RT_2]}{SD[RT_1^*] \cdot SD[RT_2^*]},$$

and with equation (13) it follows

$$\rho^* = \frac{\mathrm{cov}[RT_1, RT_2]}{\sqrt{\left(\mathrm{var}[RT_1] + \frac{m^2}{12}\right) \cdot \left(\mathrm{var}[RT_2] + \frac{m^2}{12}\right)}}. \tag{16}$$

From equation (16) one sees that ρ^* approaches to zero as m increases. As an example, assume $\mathrm{cov}[RT_1, RT_2] = -900$, $SD[RT_1] = 50$, and $SD[RT_2] = 70$. For this example one obtains for $m = 0$, 2, 4, 8, 16, 32 and 64 ms the values -0.257, -0.257, -0.257, -0.257, -0.255, -0.251 and -0.233 for ρ^*, respectively.

The relative bias of ρ^*. In general ρ^* is less affected by m if the variances of RT_1 and RT_2 are relatively large. To show this let ρ be the product moment correlation coefficient of RT_1 and RT_2 and assume that $\mathrm{var}[RT_1] = \mathrm{var}[RT_2] = k \cdot m$. Under this assumption one obtains from equation (16)

$$\frac{|\rho| - |\rho^*|}{|\rho|} = \frac{1}{12 \cdot k^2 + 1} \tag{17}$$

expressing the difference $|\rho| - |\rho^*|$ relative to $|\rho|$ as a function of k. For k equal to 1, 2, 3, 4, 5 and 6 one obtains the values 0.077, 0.020, 0.009, 0.005, 0.003 and 0.002 for this ratio.

Correction for attenuation. Fortunately it is possible to recover ρ from ρ^*. Using again equation (16) one derives

$$\rho = \rho^* \left[\left(1 - \frac{m^2}{12 \cdot \mathrm{var}[RT_1^*]} \right) \cdot \left(1 - \frac{m^2}{12 \cdot \mathrm{var}[RT_2^*]} \right) \right]^{-0.5}. \tag{18}$$

This formula should be used to compute ρ when the quantities ρ^*, $\mathrm{var}[RT_1^*]$, $\mathrm{var}[RT_2^*]$ and the clock's time resolution m are known.

3. Conclusion

The present paper investigated the relationship of measured and true RT if the accuracy of an RT clock is finite. It was found that the time resolution of an RT clock has almost no effect on detecting mean RT differences even if the time resolution is about 30 ms or worse. We provided formulae to correct means, SDs, and product moment correlation coefficients of measured RTs. Many computations with theoretical RT distributions revealed that the correction formulae worked very satisfactorily if $SD[\mathrm{RT}] \leq m$. Equations (14) and (17) can be used to judge whether the time resolution of a given clock may be sufficient for RT measurement.

The purpose of this paper should not be to motivate researchers to carry out sloppy work. However, most personal computers provide built-in timing mechanisms (e.g. a time function in BASIC) that have time resolutions between 10 and 30 ms. We believe that many experimental psychologists hesitate to use such a built-in function and therefore develop a more accurate timing mechanism at machine language level. This paper shows that such an additional effort is often unnecessary.

Implicit throughout the paper is the assumption that the RT clock always rounds up, so that any fraction of a unit is sufficient to index or activate the next unit level (e.g. 31 ms would round up to 60 ms, given $m = 30$ ms time resolution). However, it may be that several computer clocks do not work in this way, that is they may round down (e.g. 59 ms would round down to 30 ms, given $m = 30$ ms time resolution). In this case our conclusions are still appropriate with the only exception that equation (11) has to be changed to $E[\mathrm{RT}] = E[\mathrm{RT}^*] + \frac{m}{2}$.

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