JOURNAL OF MATHEMATICAL PSYCHOLOGY 29, 243–270 (1985)

The Short-Term Storage as a Buffer Memory between Long-Term Storage and the Motor System: A Simultaneous-Processing Model

ROLF ULRICH AND KLAUS DIETZ

Universität Tübingen, Federal Republic of Germany

It is assumed that in a free-recall task the short-term store serves as a memory buffer between the long-term store and a final motor stage of word production. Because of this the retrieval process in long-term store is not hindered by the final motor stage of word production since continued output from the long-term store queues in the buffer for motoric processing. Otherwise a time consuming communication between the two processes would be necessary. A stochastic model of this conceptualization is provided to predict the temporal course of free recall as well as a paradigm in which the contribution of the short-term store in free recall can be studied. The experimental results from this paradigm were used to test the model and to estimate short-term storage capacity on the basis of the time course of free recall. The model predictions were in good agreement with the data and the capacity estimate coincides well with estimates found by totally different methods reported in the literature.

1. INTRODUCTION

Two components of the human memory system have been studied extensively over the past 20 years: the short-term and the long-term stores. Information temporarily in the activated state is said to be in short-term store (STS). It is assumed that STS controls processes such as coding and rehearsal (Atkinson & Shiffrin, 1968,1971; Raaijmakers & Shiffrin, 1980, 1981; Shiffrin, 1975; Waugh & Norman, 1965). It is also assumed that STS is limited in capacity (Shiffrin, 1976), so that only a limited number of items may be retained, rehearsed, and coded at one time. The control processes in STS are believed to be prerequisites for permanent storage of information in long-term store (LTS). Accordingly, much attention has been paid to the role of the STS in transferring information to LTS for permanent

We thank Dietrich Albert, Miles Hewstone, Klaus-Peter Muthig, and especially Dirk Vorberg for many helpful suggestions. We are grateful to the Editor and an anonymous referee for many helpful comments on an earlier draft of this paper. Requests for reprints should be sent to Rolf Ulrich, Universität Tübingen, Psychologisches Institut, Friedrichstrasse 21, D-7400 Tübingen, Federal Republic of Germany.

0022-2496/85 $3.00

storage. However, there seem to be no precise attempts to account for the role of STS when information has to be retrieved from LTS.

In closing this gap the present article provides a possible contribution of the STS when subjects continually retrieve information from LTS, as is the case in free recall. In such a task subjects are instructed to recall all words of a previously learned list or from a specified taxonomic category, such as animals, vegetables, pieces of furniture, or occupations (Bousfield & Sedgewick, 1944; or for a review, Murphy & Puff, 1982). Subjects may produce the words in any order they choose.

It seems plausible that in such a task the STS may function as a memory buffer or waiting room between LTS and a final motor channel. Since subjects must verbalize their recalls, motor programs of the retrieved information must be prepared, started, and executed in a final motor channel. Retrieved information could then wait in STS for further processing if the motor channel should be busy.

By studying the temporal course of the recalled words, one may get insights as to whether STS contributes anything as a buffer memory, as demonstrated later. To this end we measure the period between the completions of any two consecutive recalls, denoted in the following as *intercompletion time*. We derive the distribution of the intercompletion time under the assumption that STS contributes as a memory buffer in free recall. The knowlegde of this distribution, for example, enables one to estimate the capacity of the STS.

We organize the article as follows: (1) In the next section the basic idea of the Simultaneous-Processing Model is considered. (2) Then two conceptually different retrieval processes in LTM are introduced. Both retrieval processes may be considered as examples. Each of them is easily incorporated into the framework of the Simultaneous-Processing Model. How these incorporations are accomplished, along with the introduction of important terms, are our main messages in this section. (3) Next, we state the specific assumptions of the model. (4) Then, the distribution of the intercompletion times is derived for the Simultaneous-Processing Model. In addition, we provide easy-to-compute expressions for the means and variances of intercompletion times. (5) In a final section, we describe an experiment giving evidence that information is stored in STS for further motor processing. Hereby, the subject may preload STS with information from LTS before the actual signal to start overt recall appears. The obtained data from this delayed recall paradigm are used to check the Simultaneous-Processing Model and to estimate the capacity of the STS.

2. The Basic Idea: Simultaneous Processing

Consider the typical temporal course of the recalled words in a free-recall task: After asking a subject to recall all words from a previously learned list, word after word is retrieved from LTS and pronounced. At the beginning the words follow in rapid succession. After several words have been recalled the time between two consecutive recalled words increases.

The speed at the beginning of the recall is somewhat surprising if one takes into account the time to prepare speech, to activate the muscles, and to pronounce a recalled word. This recall speed would surely be impossible if the retrieval of a unit in LTS and its word production were two strictly serial processes, that is, while the word production process runs, the retrieval process in LTS rests and vice versa. Instead it is more probable that neither process is interrupted while the other one runs: Retrieval of new units in LTS and word production can go simultaneously and almost independently. Under these simultaneous-processing assumptions the rapid retrieval process is not hindered by slow motor processing of a previously retrieved word.

However, what happens when a word in LTS is retrieved while the motor process is busy with a previously retrieved word? There is no problem in answering this question if one postulates that the STS serves as a buffer memory between the LTM and the motor stage. The STS receives units from the LTM and makes them available for the final motor stage. The retrieval process can put the next unit into the buffer immediately after it finishes putting the current one there irrespective of whether the motor process runs it immediately or not.

In contrast, if there was no buffer memory between the motor stage and the LTM, the two processes would have to be closely synchronized, the LTM passing the next unit to the motor stage whenever the latter was finished with the current one. This might require time consuming communication of the two processes which is not needed if a memory buffer were available.

3. Simultaneous-Processing Model and Possible Retrieval Processes in LTS

As noted, we are primarily concerned with the contribution of the STS as a memory buffer in free recall. Therefore, the specification of any retrieval process in LTS is only of minor interest within the context of the present work. Any retrieval process may be considered within the framework of the Simultaneous-Processing Model as long as the time between two consecutive outputs from LTS is *exponentially* distributed (the requirement of this assumption will become clearer when dealing with the mathematics of the model).

In the following, we describe two conceptually different retrieval processes which might be considered as suitable candidates for the Simultaneous-Processing Model. It should be stressed that these two retrieval processes are simplifications, neglecting many well-established findings indicating the importance of organizational factors in LTS which are well dealt with by a large-scale theory such as that of Raaijmakers and Shiffrin (1980, 1981). However, we think that these two retrieval processes are useful for introducing the terminology with more distinctness, making the matter more concrete, and also, that they are helpful in showing how to incorporate a particular retrieval process into the framework of the Simultaneous-Processing Model.

3.1. *Random-Search Model*

The Random-Search Model (McGill, 1963) assumes that the subject restricts the search in LTS to a limited area (e.g., all animals) if the subject is instructed to name as many members from a specified taxonomic category (e.g., four legged animals) as can be remembered. The limited area consists of n relevant items (e.g., dog, horse, mouse, etc.) and of an unspecified number of irrelevant items (e.g., snake, fish, bird, etc.). The retrieval of the relevant items from the limited area is analogous to a sampling-with-replacement process: Item after item is randomly sampled from the limited search area. After every sampling the corresponding item is inspected to see whether it is relevant (that is, whether it is a member from the specified category) *and* whether it has not been recalled before. If a sampled item is relevant and has not been recalled before it will be copied. The original item is marked as recalled and replaced in the limited area of the LTS. (Note that an already sampled item could in principle be sampled more than once, because items are replaced in the limited area after inspection. This aspect of the Random-Search Model is very appealing, since introspection reveals that an item might be remembered more than once during the course of a single free-recall trial.)

Now, the copy of the just retrieved item is transferred to STS. There it joins the waiting line consisting of copies arrived previously, and waits for its entrance into the motor channel. After the copy has entered the motor channel, the corresponding motor program for its verbalization is prepared and executed. The motor channel is freed from the copy as soon as the present recall is finished. The elapsed time beginning with the copy's entrance into the motor channel until its overt recall is denoted as the *motor service time.* It should be stressed that we are not constrained to assume that the queue in STS is handled in the order first come–first served; pushing forward may be allowed in STS!

Let us now draw attention to the *interarrival time density.* The time between the arrivals of the $(i-1)$st and ith copy at the STS is denoted as the *interarrival time* D_i $(i = 1,..., n)$ and the corresponding density as $f_{D_i}(t)$.

McGill (1963, pp. 343–344) showed for the Random-Search Model that if the individual sampling times are exponentially distributed with parameter s then the ith interarrival time density of D_i also has an exponential distribution

$$f_{D_i}(t) = a_i \exp(-a_i t) \tag{1}$$

with rate $a_i = sg(n-i+1)$. Here, g is the probability of sampling a relevant item in the search area, as defined before. The same interarrival time density is obtained if one assumes that the individual sampling times are constant and very small (Albert, 1968, 1972).

Before proceeding with the second retrieval process some comments are in order about the one just described: (1) It is assumed that every relevant item in the limited area has the same probability g of being sampled. This assumption is not very cogent, since some items are more salient (e.g., dog) and therefore produced earlier than others (e.g., weasel). A generalization of the Random-Search Model in this direction has recently been undertaken by Vorberg and Ulrich (1985). (2) It is assumed that the search continues until all n relevant items in the limited area have been retrieved. Various stopping rules based on the number of irrelevant samplings are proposed by Schulz and Albert (1976) and may be used as modifications.

3.2. *Parallel-Activation Model*

The Random-Search Model assumes that the retrieval process in LTS is strictly serial: Only one item could be sampled and inspected at any time. In contrast to this serial retrieval process one may assume that all n relevant items are simultaneously activated. However, an item is not immediately retrievable because it takes a random delay before it is sufficiently activated and its copy finished for taking it into STS. This Parallel-Activation Model is more compatible with recent parallel conceptualiziations about retrieval processes in LTS (cf. Hinton & Anderson, 1981; Ratcliff, 1978).

If we assume that the random delays of these relevant items are independently and exponentially distributed, having the same rate sg, then the Parallel-Activation Model is indistinguishable from the Random-Search Model, as McGill (1963, p. 347) pointed out: The interarrival time density for D_i is the same as the one for the Random-Search Model, that is, identical to Eq. (1).

One may argue with good reasons that some items are more salient and therefore the rates of the delays do differ. Indeed, a study of Kaplan and Carvellas (1969) strengthens this assumption. They have measured the time of recall for individual words and found that more salient items do have shorter recall times. Vorberg and Ulrich (1985) have recently generalized the Parallel-Activation Model to account for such findings. Their generalization may be used to improve the current version of this retrieval process.

4. Specific Assumptions of the Simultaneous-Processing Model

Before dealing with the mathematics of the Simultaneous-Processing Model we briefly outline all specific assumptions. They are sometimes simplifying assumptions so as to render the mathematics of the model tractable.

Assumption 1. The time between the arrivals of the $(i-1)$st and ith copy at the STS is denoted by D_i. Any retrieval process may be considered as long as the interarrival times D_i are independent exponentially distributed random variables having the individual rates $a_i (i = 1,..., n)$.

One may well ask whether the durations of psychological processes are well approximated by exponential random variables. Some recent evidence suggests that this is indeed the case (Ashby, 1982; Ashby & Townsend, 1980; Kohfeld, Santee & Wallace, 1981), at least for certain mental processes.

Assumption 2. The time a single copy occupies the motor channel is denoted as the motor service time M. Any distribution may be selected for M. It is assumed that the distribution of M neither depends on the number of waiting copies nor on the number of retrieved items, and that D_i and M are independent.

Assumption 3. Maximally c copies can wait in STS; that is, STS is assumed to be limited in capacity. The queue in STS need not to be served in a strict order by the motor channel — any waiting copy may be picked for motor processing.

Assumption 4. No more than one copy can be served by the motor channel at any one point of time.

Assumption 5. It may be that STS is totally occupied with copies and no further copy can be picked up. There are many possibilities in such a case: The retrieval process may be interrupted and restarted as soon as one occupied slot is freed in STS. Another possibility would be that the retrieval process is not interrupted but that the sampled items are replaced in the LTS search area without being marked as recalled. The second possibility would be attractive if restarting the retrieval process is a time-consuming process. These two possibilities are thoroughly discussed in the note at the end of the article.

Irrespective of what may happen with the retrieval process if the STS is totally occupied, the following remark applies: Let us assume for a moment that the $(i-1)$st copy enters STS and occupies the last free slot. The ith arrival cannot occur before at least one slot is freed in STS. Now consider the time elapsing between the event when one slot is freed until the arrival of the ith copy. Assumption 5 states that this time should correspond to D_i. How one may justify this assumption is explained in the note at the end of this article.

After having introduced the assumptions of the Simultaneous-Processing Model we proceed to derive the distributions of the observable intercompletion times in the next sections.

5. Intercompletion Times Predicted by the Simultaneous-Processing Model

In the subsequent sections we are concerned with the intercompletion time $C_i (i=1,..., n)$, that is, the observable time between the completions of the $(i-1)$st and ith overt recall. We derive the probability density function (pdf), mean, and variance for C_i within the framework of the Simultaneous-Processing Model.

At this point an anticipation of the main result may facilitate the understanding of the arrangement for the following sections. The main result is that the pdf $f_{C_i}(t)$ of intercompletion time C_i can be described as a probability mixture of the motor-service time M and the sum $M+R_i$ composed of motor-service time M and a random variable R_i:

$$f_{C_i}(t) = f_M(t) \cdot (1-p_i) + f_{M+R_i}(t) \cdot p_i.$$

Hereby, p_i denotes the probability that STS is empty after completing the $(i-1)$st recall, or in other words, that no copy is waiting in STS after the $(i-1)$st overt recall is finished. If STS is empty after completing the $(i-1)$st overt recall, then R_i is the time elapsing between the completion of the $(i-1)$st recall and the arrival of the ith copy at STS from LTS. Note that R_i must be shorter than the interarrival time D_i — therefore, R_i is called the residual-interarrival time.

Although so far we may not be very familiar with the above notation, this probability mixture has an intuitive appeal: If there is at least one waiting copy in STS while completing the $(i-1)$st recall, then the following intercompletion time C_i just reflects the motor-service time M for the ith copy. This event occurs with probability $(1-p_i)$. On the other hand, it may occur with probability p_i that no further copy did arrive in the meanwhile, and thus, the STS is empty when the $(i-1)$st recall is completed. In such a case one has to wait an additional time R_i for the arrival of the ith copy from LTS before motor processing of the ith copy can begin. Hence, under this condition, the intercompletion time C_i reflects the sum $M+R_i$. These considerations provide the starting point for the following analysis: How can one compute the probability p_i? The next four sections address this question.

5.1. *The Transition Diagram and Permissible Pathways*

For the purpose of the desired analysis it may be helpful to look at the free-recall process in more detail within the Simultaneous-Processing Model. This is facilitated if one sketches the course of the free-recall process by using a *transition diagram.* Such a diagram is displayed in Fig. 1 for the special case of $n=6$ relevant items and

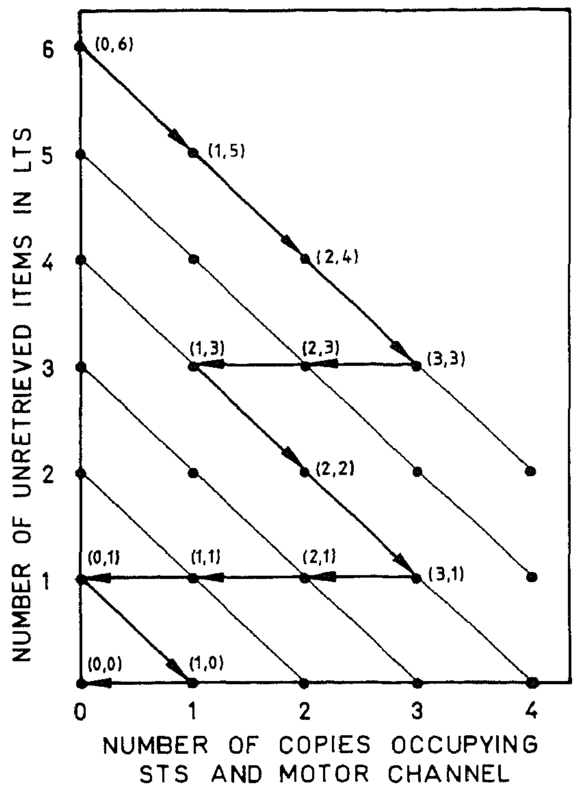

FIG. 1. Transition diagram. Filled circles represent different coordinates (o, r) of the process. Arrows describe a sample pathway.

an STS capacity of $c=3$. In the transition diagram, the horizontal axis represents the total number o of copies occupying STS and motor channel. For example, $o=4$ means that there are three waiting copies in STS and one is in the motor channel. Since at one moment only one copy could be in the motor channel, which is filled automatically as soon as a copy enters the empty STS, the number $u=o-1$ $(o \geqslant 1)$ represents the actual number of waiting copies in STS. The vertical axis represents the number r of not as yet retrieved relevant items. Thus, the coordinates (o, r) $(r=0, 1,..., n;\ o=0, 1,..., c+1;\ o+u \leqslant n)$ in the transition diagram describe the various nodes the process may visit during the course of free recall. For example, the coordinate (2, 3) means that there are still three unretrieved items in LTS and at the very moment one copy is waiting in STS and a further one is served by the motor channel. Note that the process always starts at $(0, n)$ and finishes at (0, 0). Every permissible route between start and finish describes a particular realization of the process and is called a *pathway*.

To characterize the features of permissible pathways we consider the particular one sketched in Fig. 1: After starting the process one item is retrieved in LTS and its copy is transferred to STS and passed on to the motor channel where its motor processing begins immediately. This event corresponds to the transition from coordinate (0, 6) to (1, 5). Before the motor process for the first copy has been completed, two further copies arrive at STS. Therefore, the pathway runs through the coordinates (2, 4) and (3, 3). Note that the first recall could not yet have been completed, since the motor channel is still occupied by the first copy. From (3, 3) the pathway leads to (2, 3). This section corresponds to the action of freeing the motor channel from the first copy before any further copy arrives at STS. Therefore, as soon as the process reaches the coordinate (2, 3) we can observe the completion of the first recall.

Now the motor channel is immediately occupied with a waiting copy from STS. Since no further copy from LTS arrives at the STS while the motor channel is busy with the second recall, the pathway makes a second step to the left and we observe the completion of the second recall as soon as the process arrives at (1, 3). The one copy waiting in STS is now immediately transferred to the motor channel. After the second recall, two further copies enter STS before completion of the third recall is observable. The third recall is completed as soon as the process visits (2, 1). While the retrieval process is going on for the last item in LTS, the two waiting copies are being served one by one, and thus, the coordinate (0, 1) is visited. The last copy enters the empty STS and is immediately served; its recall is completed as soon as the process reaches (0, 0).

One might have noticed that the pathway is constrained to use one of two directions at each coordinate: First, whenever a copy enters STS, the pathway moves one step southeast. Second, whenever the motor processing of a copy is completed the pathway moves one step to the west. That is, at the end of a single step to the west we can observe the completion of overt recall for a copy that was just before in the motor channel. We should keep this property of the transition diagram in mind when deriving the desired probability p_i in the next three sections.

5.2. *Pathway Sections and Intercompletion Times*

It is the purpose of this section to describe those pathway sections in the transition diagram which are associated with intercompletion time C_i. There are several pathway sections which may lead to an observed intercompletion time. All those pathway sections associated with C_i have to be defined for further analysis. This task is performed with the aid of the transition diagram in Fig. 1 before presenting a general definition: Visualize in the above example the possible states the process might be visiting while one observes the intercompletion time C_3. For the particular pathway sketched in Fig. 1 we would start the clock for timing C_3 when the process reaches the coordinate (1, 3), because this would correspond to the completion of the second recall. When the process arrives at (2, 1) we should immediately stop the clock, because this coordinate corresponds to the completion of the third recall. Thus, the particular pathway section underlying C_3 in this example can be described by a *starting* and an *end state.* However, as mentioned before, there are many pathway sections and each of them may in principle generate an observed intercompletion time.

To describe all sections generating C_3 we introduce the sets U and V containing starting and end states of all possible pathway sections, respectively. The states in U and V are found on the straight lines having a slope of minus one and passing through coordinates (0, 4) and (0, 3), respectively. Thus, we have the set $U = \{0, 1, 2, 3\}$ and the set $V = \{0, 1, 2, 3\}$. An element u of U (v of V) represents the total number of copies occupying STS and motor channel immediately after the completion of the $(i-1)$st recall (ith recall). The particular pathway section in Fig. 1 generating C_3 has starting state $u = 1$ and end state $v = 2$.

Now let $U \times V$ be the Cartesian product of the sets U and V and let (u, v) be an element out of $U \times V$.[1] Determine all those elements in $U \times V$ which are pathway sections, that is, all those elements which may in principle generate intercompletion time C_3. Denote this set of all possible pathway sections by A_3. In order to be a member of the set A_3, an element (u, v) out of $U \times V$ must satisfy the condition $v \geqslant \text{MAX}(u-1, 0)$. This condition follows from the fact that a pathway at any state can either continue to the west or to the southeast. In Fig. 2 the sets of possible pathway sections $A_1, \dots, A_6$ for the corresponding intercompletion times $C_1, \dots, C_6$ of the above example are displayed. Note that A_i depends on the recall number i.

We now generalize the above considerations to characterize the sets U, V, and A for arbitrary numbers n of relevant items, STS capacity c, and recall number i.

DEFINITION. Any pathway section associated with intercompletion time C_i $(i = 1, \dots, n)$ is defined by its starting state u and its end state v, with $u \in U_i$ and $v \in V_i$. The elements of U_i are found on the straight line passing through coordinate $(0, n-i+1)$ and having a slope of minus one. The elements of V_i are also found on

[1] One should not confuse the ordered pairs (o, r) and (u, v). The first ordered pair denotes the coordinates o and r in the transition diagram and the second one a particular pathway section with starting state u and end state v.

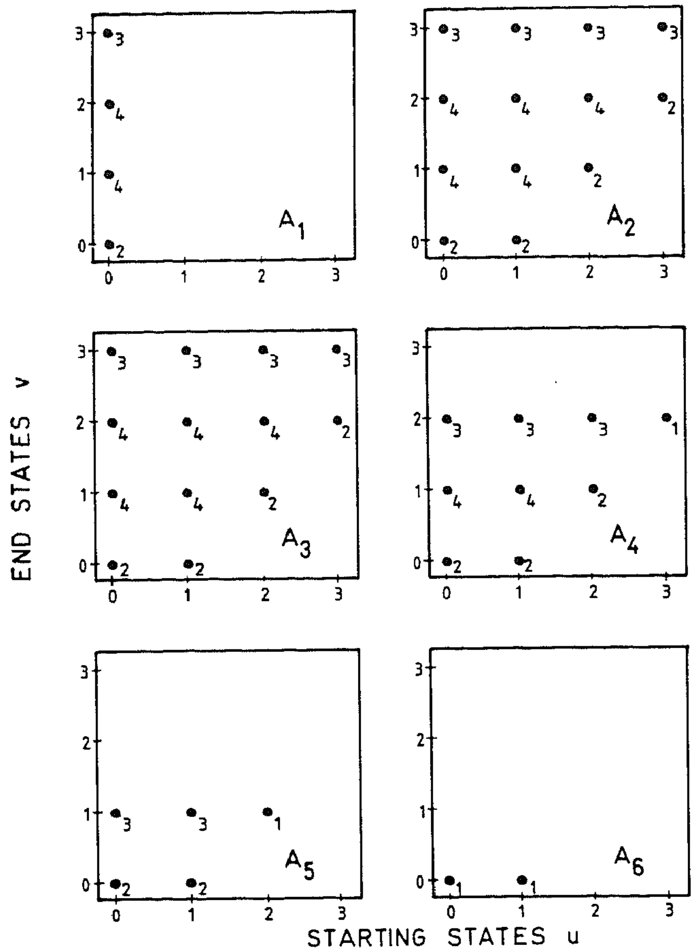

FIG. 2. The six patches show the sets $A_1, \ldots, A_6$ for the above example ($n=6$, $c=3$). A filled circle represents a pathway section (u, v) out of A_i which may generate intercompletion time $C_i (i=1, \ldots, 6)$. A pathway section is defined by its starting state u and its end state v. The digit beside the filled circle denotes the subset membership of each pathway section within a particular A_i. The subset membership is used for computing the transition probability $P_i(v, u)$ introduced in Section 4.3.

a straight line which is parallel to the one of U_i and passes through $(0, n-i)$. Hence, $U_i = \{u\colon u = 0, 1, \ldots, \mathrm{MIN}(c, n-i+1)\}$ and $V_i = \{v\colon v = 0, 1, \ldots, \mathrm{MIN}(c, n-i)\}$ describe the possible number of copies occupying STS at the very moment the $(i-1)$st and ith overt recalls are completed, respectively. In addition, since the process always begins at coordinate $(0, n)$, we have $U_1 = \{0\}$. And finally, the set A_i of all pathway sections which may generate intercompletion time C_i is a subset of the Cartesian product $U_i \times V_i$ and can be expressed as

$$\begin{aligned} A_i &= \{(u, v)\colon u=0; v=0, 1, \ldots, \mathrm{MIN}(c, n-1)\} \qquad \text{for} \quad i=1 \\ &= \{(u, v)\colon u=0, 1, \ldots, \mathrm{MIN}(c, n-i+1); \\ &\qquad v = \mathrm{MAX}(u-1, 0), \ldots, \mathrm{MIN}(c, n-i)\} \qquad \text{for} \quad i>1. \end{aligned} \tag{2}$$

5.3. *Transition Probabilities*

In this section we compute the transition probability $P(Y_{i+1} = v \mid Y_i = u)$ that a pathway section out of A_i starting at u will have the end state v. Y_i and Y_{i+1} are random variables representing the starting and end states, respectively. For reasons of notational simplicity, we sometimes abbreviate $P(Y_{i+1}=v \mid Y_i=u)$ by $P_i(v, u)$, and all transition probabilities are defined to be zero if (u, v) is not contained in A_i. It is also helpful to notice that $P_i(v, 0)=P_i(v, 1)$ since the transition from $(0, i)$ to $(1, i-1)$ occurs with probability one. Therefore, we can leave out of consideration all those pathway sections starting with $u=0$ when deriving the transition probabilities.

It is appropriate for deriving the desired transition probabilities to partition the whole set A_i into four disjoint subsets. All pathway sections in each subset share the same common features for deriving their transition probabilities. In Fig. 2 we have marked these four subsets within each A_i $(i=1,..., 6)$ for the above example. Note hereby that for a given end state v, the corresponding pathway sections $(0, v)$ and $(1, v)$ are members of the same subset since as mentioned before $P_i(v, 0)=P_i(v, 1)$. We now describe the computation of the transition probabilities for the four subsets.

Subset 1: $\{(u, v)\colon v=u-1 \text{ and } u=n-i+1\}$. The pathway sections in this subset occur if all n items in LTS have been retrieved, but some of their copies still wait in STS for motor processing. Thus, the pathway sections for Subset 1 are found on the horizontal axis in the transition diagram. Since all items have been retrieved in LTS, the process is constrained to walk to the west, with $P_i(v, u)=1$.

Subset 2: $\{(v, u)\colon v=u-1 \text{ and } u<n+1-i\}$. The common feature of all pathway sections in Subset 2 is as before: Again we deal with one-step transitions to the west. However, in contrast to Subset 1, there is at least one unretrieved item in LTS while this step occurs. In the above example, the step from state $(2, 3)$ to $(1, 3)$ would be a pathway section belonging to Subset 2.

We describe this situation more formally: At first we note that there are u copies in STS after the $(i-1)$st recall is completed. Therefore, the STS is waiting for the arrival of the next copy from LTS which must be the $(i+u)$th arrival. Second, let R_{i+u} be the residual-interarrival time of the $(i+u)$th copy, that is the time elapsing since the completion of the $(i-1)$st recall until the arrival of the $(i+u)$th copy at STS (Fig. 3). Since motor processing of the ith recall is completed before the $(i+u)$th copy arrives, the one-step pathway section from u to $v=u-1$ is equivalent to the occurrence of the event $\{M<R_{i+u}\}$.

PROPOSITION 1. *The transition probability for any pathway section* (u, v) *belonging to Subset* 2 *is*

$$P_i(u, v)=E(e^{-a_{i+u}M})=\rho(a_{i+u}),$$

where ρ *denotes the Laplace transform of the motor-service time* M, *and* a_{i+u} *is the exponential parameter of the interarrival time* D_{i+u}.

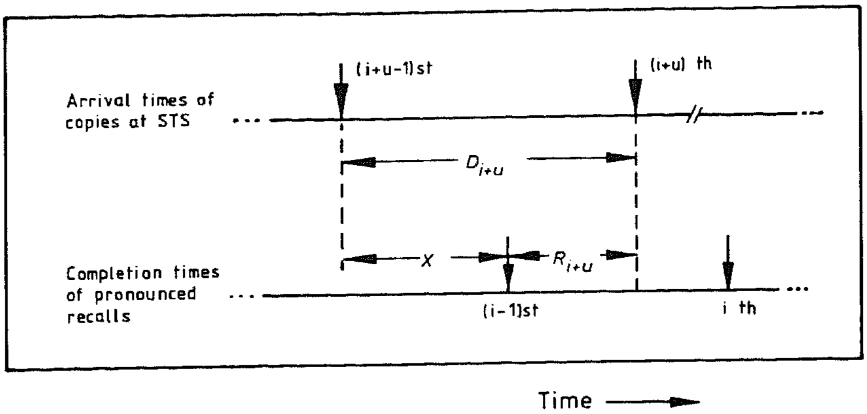

FIG. 3. Definition of residual-interarrival time R_{i+u}. Upper line shows the arrival times of the $(i+u-1)$st and $(i+u)$th copy at STS. Lower line shows the point of times when completing the $(i-1)$st and ith recall. In the Appendix it is shown that R_{i+u} is exponentially distributed if the interarrival time D_{i+u} is also exponentially distributed. This result holds for any distributional assumption about M. The variable X as defined here is used in the Appendix.

Proof.

$$\begin{aligned} P_i(v, u) &= P(R_{i+u} > M) \\ &= \int_0^\infty \int_m^\infty f_{M,R_{i+u}}(m, r)\, dr\, dm \\ &= \int_0^\infty f_M(m) \left(\int_m^\infty f_{R_{i+u}}(r)\, dr \right) dm \\ &= \int_0^\infty f_M(m)(1 - F_{R_{i+u}}(m))\, dm. \end{aligned}$$

In the Appendix it is shown that the residual-interarrival time R_{i+u} has a distribution identical to that of the interarrival time D_{i+u} if the interarrival times are exponentially distributed. Hence, the expression in bracket is $\exp(-a_{i+u} \cdot m)$.

$$\begin{aligned} P_i(v, u) &= \int_0^\infty f_M(m) \exp(-a_{i+u} \cdot m)\, dm \\ &= E(e^{-a_{i+u}M}). \end{aligned}$$

The last expression is the Laplace transform $\rho(a_{i+u})$ of the motor-service time M. This completes the proof.

Subset 3: $\{(u, v)$: $v = \text{MIN}(c, n-i)$ *and* $u \leqslant n-i\}$. First, we consider those pathway sections in Subset 3 for which the end state is $v = c$. In the above example a pathway section with $v = 3$ and $u = 1$ generating C_3 belongs to this category (Fig. 1). What must happen for a pathway section with any starting state u to end at $v = c$?

After completing the $(i-1)$st recall the next copy immediately enters the motor channel. Before the motor channel is freed from this copy, one after the other arrives at STS until STS is totally occupied. Or in other words, motor processing of the ith copy required so much time that STS had been totally filled up with copies from LTS.

In order to describe this event formally, we define the random variable $S_{i,u,v}$ which represents the elapsed time from completion of the $(i-1)$st recall until the arrival of the $(i+v)$th copy at STS. Note that $S_{i,u,v}$ can be expressed as a sum having the components $R_{i+u}, D_{i+u+1}, \ldots, D_{i+v}$. Two examples are $S_{3,2,4} = R_5 + D_6 + D_7$ and $S_{3,2,2} = R_5$.

Now $S_{i,u,c}$ represents the interval between the completion of the $(i-1)$st copy until the arrival of the $(i+c)$th copy at STS. Therefore, if the motor-service time M for the ith copy is greater than $S_{i,u,c}$, the above described event occurs. Hence, the transition probability of a pathway section in Subset 3 with end state c must be equal to the probability of the event $\{M > S_{i,u,c}\}$.

The alternative case, $v = n - i$, has formally the same consequence as the just discussed one of $v = c$. The only difference is that STS can not be filled up because the number of relevant items in LTS is now insufficient. The pathway section (1, 2) generating C_4 in the above example belongs to this category (Fig. 1). An end state of $v = n - i$ occurs if the nth (the last one) copy has entered STS before the ith recall is completed, that is, if the event $\{M > S_{i,u,n-i}\}$ has occurred.

In summary, the transition probability for any pathway section out of Subset 3 must be equal to the probability of the event $\{M > S_{i,u,v}\}$.

PROPOSITION 2. *The transition probability for any pathway section* (u, v) *belonging to Subset* 3 *is*

$$P_i(v, u) = 1 - \sum_{k=u+i}^{v+i} C_{kivu} \cdot \rho(a_k)$$

with

$$C_{kivu} = \prod_{\substack{j=u+i \\ j \neq k}}^{v+i} 1/(1 - a_k/a_j).$$

Proof. $P_i(u, v) = P(M > S_{i,u,v})$. Let $F_{S_{u,v,i}}(t)$ be the distribution function of the sum $S_{u,v,i}$, and note the similarity of the proof here to the one of Proposition 1

$$P_i(v, u) = \int_0^\infty f_M(t) \cdot F_{S_{u,v,i}}(t)\, dt.$$

Since the individual interresponse times are assumed to be exponentially distributed, $F_{S_{u,v,i}}(t)$ corresponds to the general Erlangian distribution (cf. McGill & Gibbon, 1965; Townsend & Ashby, 1983, p. 209)

$$F_{S_{u,v,i}}(t) = 1 - \sum_{k=u+i}^{v+i} C_{kivu} \cdot \exp(-a_k \cdot t), \qquad t > 0, \tag{3}$$

and C_{kivu} is the constant defined in Proposition 2.

To continue the proof, we insert (3) for $F_{S_{u,v,i}}(t)$:

$$P_i(v, u) = 1 - \int_0^\infty f_M(t) \cdot \left(\sum_{k=u+i}^{v+i} C_{kivu} \cdot \exp(-a_k t) \right) dt$$
$$= 1 - \sum_{k=u+i}^{v+i} C_{kivu} \cdot \int_0^\infty f_M(t) \cdot \exp(-a_k t)\, dt.$$

The integral is equal to the Laplace transform of M. The proof is finished.

Subset 4: $\{(u, v)$: $u \leqslant v \leqslant \mathrm{MIN}(c-1, n-i-1)\}$. The remaining pathway sections belonging to A_i are the most complicated ones to describe formally: Immediately after completing the $(i-1)$st recall, the motor channel is occupied with a copy from STS. Until the completion of the ith recall at least one copy arrives. In contrast to Subset 3, the ith recall is completed before STS is filled up or before all relevant items in LTS have been found.

In the above example, the pathway section starting at coordinate (1, 3) and ending at (2, 1) belongs to Subset 4. Two further copies entered STS between the completions of the second and third recall. Therefore, this pathway section passed the coordinates (2, 2) and (3, 1). In order to reach the coordinate (3, 1), the event $\{M > R_4 + D_5\}$ must have occurred. Since the pathway section moves to the west at (3, 1), motor processing of the third recall finishes before the arrival of the sixth copy from LTS, that is, M must be smaller than the sum $R_4 + D_5 + D_6$. Therefore, this particular pathway section corresponds to the occurrence of the event $\{R_4 + D_5 + D_6 > M > R_4 + D_5\}$.

It follows from the considerations here that the transition probability for any pathway section (u, v) contained in Subset 4 is equal to the probability of the event $\{S_{i,u,v,} + D_{v+i+1} > M > S_{i,u,v}\}$, where $S_{i,u,v}$ represents the sum defined before.

PROPOSITION 3. *The transition probability $P_i(u, v)$ of pathway sections (u, v) belonging to Subset* 4 *is*

$$P_i(u, v) = \sum_{k=u+i}^{v+i} C_{kivu} \cdot 1/(1 - a_{v+i+1}/a_k) \cdot (\rho(a_{v+i+1}) - \rho(a_k)),$$

where C_{kivu} is given in Proposition 2.

Proof. To simplify matters, we abbreviate $S = S_{u,v,i}$ and $D = D_{v+i+1}$. Consequently,

$$P_i(v, u) = P(S < M < S + D)$$
$$= \int_0^\infty \int_0^m \int_{m-s}^\infty f_{S,M,D}(\mathrm{s}, m, t)\, dt\, ds\, dm.$$

$f_{S,M,D}(s, m, t)$ denotes the joint density of S, M, and D. Since independence is assumed,

$$P_i(v, u) = \int_0^\infty \int_0^m \int_{m-s}^\infty f_S(s)\, f_M(m)\, f_D(t)\, dt\, ds\, dm$$

$$= \int_0^\infty f_M(m) \int_0^m (1 - F_D(m-s))\, f_S(s)\, ds\, dm.$$

We insert the pdf of $F_S(s)$ for $f_S(s)$. This pdf is given by differentiating (3). By assumption, the expression in the bracket equals $\exp(-a_{v+i+1} \cdot (m-s))$. Thus,

$$P_i(v, u) = \int_0^\infty f_M(m) \int_0^m \exp(-a_{v+i+1} \cdot (m-s)) \times \left(\sum_{k=u+i}^{v+i} C_{kivu} \cdot a_k \cdot \exp(-a_k s) \right) ds\, dm$$

$$= \int_0^\infty f_M(m) \exp(-a_{v+i+1} m) \sum_{k=u+i}^{v+i} C_{kivu} \cdot a_k \times \int_0^m \exp(-(a_k - a_{v+i+1}) s)\, ds\, dm$$

$$= \int_0^\infty f_M(m) \left(\sum_{k=u+i}^{v+i} C_{kivu} \cdot a_k \cdot \frac{\exp(-a_{v+i+1} m) - \exp(-a_k m)}{a_k - a_{v+i+1}} \right) dm$$

$$= \sum_{k=u+i}^{v+i} C_{kivu} \cdot 1/(1 - a_{v+i+1}/a_k) \times \left(\int_0^\infty f_M(m) \exp(-a_{v+i+1} m)\, dm - \int_0^\infty f_M(m) \exp(-a_k m) \right) dm.$$

The integrals are Laplace transforms of M. The proof is complete.

It should be pointed out that for computing the transition probabilities, one only needs the Laplace transform of the motor-service time M. Many well-known densities which may be used for the pdf of M possess Laplace transforms which are easy to compute.

Before concluding this section we summarize the results for later reference: For any pathway section (u, v) out of A_i the transition probability $P(Y_{i+1} = v \mid Y_i = u)$ is

$$P_i(v, u) = 1, \qquad \text{for} \quad v = u - 1 \text{ and } u = n - i + 1; \text{ or } c = 0$$

$$= \rho(a_{u+i}), \qquad \text{for} \quad v = u - 1 \text{ and } u < n - i + 1$$

$$= 1 - \sum_{k=u+i}^{v+i} C_{kivu} \cdot \rho(a_k), \qquad \text{for} \quad v = \mathrm{MIN}(c, n-i) \text{ and } u \leqslant n - i$$

$$= \sum_{k=u+i}^{v+i} C_{kivu} \cdot 1/(1 - a_{v+i+1}/a_k) \cdot (\rho(a_{v+i+1}) - \rho(a_k)), \qquad \text{elsewhere}$$

with

$$C_{kivu} = \prod_{\substack{j=u+i \\ j \neq k}}^{v+i} 1/(1 - a_k/a_j). \tag{4}$$

Note in using (4) that $P_i(v, 0) = P_i(v, 1)$. Hence, $P_i(v, 0)$ does not need to be computed in addition.

5.4. *State Probabilities*

So far, all probabilities we have dealt with are conditional probabilities of the stochastic process $\{Y_i; i = 1,..., n\}$. We now specify the unconditional probabilities $P(Y_i = u)$, $u \in U_i$, called state probabilities. For instance, $P(Y_i = u)$ is the probability that u copies occupy STS at the very moment when the $(i-1)$st recall is completed. Of special interest is the probability $P(Y_i = 0)$ for deriving the pdf of the intercompletion time C_i in the next section.

To start with, the state probabilities are expressed as a row vector, denoted here by **s**, subcripted with the recall number i

$$\mathbf{s}_i = [P_i(0), P_i(1),..., P_i(c)],$$

where $P_i(u)$ is an abbreviation for $P(Y_i = u)$. Since, the process always starts at $(0, n)$, the initial vector is given by $\mathbf{s}_1 = (1, 0, 0, 0,...)$.

The transition probabilities $P(Y_{i+1} = v \mid Y_i = u)$ given by (4) form the $(c+1) \times (c+1)$ transition matrix $\mathbf{T}_i$

$$\mathbf{T}_i = \begin{bmatrix} P_i(0, 0) & P_i(1, 0) & P_i(2, 0) & \cdots & P_i(c, 0) \\ P_i(0, 1) & P_i(1, 1) & P_i(2, 1) & \cdots & P_i(c, 1) \\ 0 & P_i(1, 2) & P_i(2, 2) & \cdots & P_i(c, 2) \\ 0 & 0 & P_i(2, 3) & \cdots & P_i(c, 3) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & P_i(c, c) \end{bmatrix}.$$

Elements of $\mathbf{T}_i$ for which $(u, v) \notin A_i$ are defined to be zero.

The multiplication of $\mathbf{s}_i$ by $\mathbf{T}_i$ gives $\mathbf{s}_{i+1}$, by definition,

$$\begin{aligned} \mathbf{s}_i\mathbf{T}_i &= \left[\sum_u P_i(0, u)P_i(u), \sum_u P_i(1, u)P_i(u), \cdots, \sum_u P_i(c, u)P_i(u)\right] \\ &= [P_{i+1}(0), P_{i+1}(1), \cdots, P_{i+1}(c)] \\ &= \mathbf{s}_{i+1}. \end{aligned}$$

Therefore, the equation

$$\mathbf{s}_{i+1} = \mathbf{s}_i\mathbf{T}_i \tag{5}$$

together with the initial vector $\mathbf{s}_1$ provides the desired state probabilities. For further derivations the state probability $P_i(0)$ is most important.

5.5. *Intercompletion-Time Distribution*

As we stated earlier the pdf $f_{C_i}(t)$ of the intercompletion time C_i can be described by a probability mixture. To recapitulate, given that the STS is not empty after the $(i-1)$st recall then the intercompletion time C_i reflects just the motor-service time M. This event $\{Y_i>0\}$ has probability $P(Y_i>0)=1-P(Y_i=0)$ which is obtained via the state vector $\mathbf{s}_i$. On the other hand, if there is no copy in STS after completing the $(i-1)$st recall, one has to wait until the arrival of the ith copy at STS and the time it spends in the motor channel. The elapsed time from completing the $(i-1)$st recall until the arrival of the ith copy at STS corresponds to the residual time R_i which is exponentially distributed with rate a_i (see the Appendix). Thus, for the event $\{Y_i=0\}$ we have $C_i=M+R_i$.

PROPOSITION 4. *The pdf of intercompletion time* C_i *is*

(i) $f_{C_i}(t)=f_{C_i}(t \mid Y_i>0)\cdot P(Y_i>0)+f_{C_i}(t \mid Y_i=0)\cdot P(Y_i=0)$

(ii) $f_{C_i}(t \mid Y_i>0)=f_M(t)$

(iii) $f_{C_i}(t \mid Y_i=0)=f_M(t) * f_{R_i}(t)$

Proofs. (i) Obvious, since $C_i=R_i+M$ $(C_i=M)$ if no (at least one) copy waits in STS at the very moment when completing the $(i-1)$st recall;

(ii) since C_i must be identical to M if $Y_i>0$, and because of Assumption 2 (see Sect. 4);

(iii) since R_i and M are assumed to be dependent.

We now turn to the mean and variance of C_i:

PROPOSITION 5. *Let* $E(M)$ *be the mean motor-service time and* $p_i=P(Y_i=0)$. *Then*

$$E(C_i)=E(M)+p_i/a_i.$$

Proof.

$$\begin{aligned} E(C_i) &= E[E(C_i \mid Y_i)] \\ &= E(C_i \mid Y_i=0)\cdot p_i+E(C_i \mid Y_i>0)\cdot(1-p_i) \\ &= [E(M)+E(R_i)]\cdot p_i+E(M)\cdot(1-p_i) \\ &= E(M)+E(R_i)\cdot p_i. \end{aligned}$$

Since $E(R_i)=1/a_i$, the proof is completed.

PROPOSITION 6. *Let* Var(M) *be the variance of the motor-service time. Then*

$$\mathrm{Var}(C_i)=\mathrm{Var}(M)+p_i(2-p_i)/a_i^2.$$

Proof. $\operatorname{Var}(C_i) = E(C_i^2) - [E(C_i)]^2$. $E(C_i)$ is given by Proposition 5. So, we need a closer look at $E(C_i^2)$

$$\begin{aligned}
E(C_i^2) &= E[E(C_i^2 \mid Y_i)] \\
&= E(C_i^2 \mid Y_i = 0) \cdot p_i + E(C_i^2 \mid Y_i > 0) \cdot (1 - p_i) \\
&= \{\operatorname{Var}(C_i \mid Y_i = 0) + [E(C_i \mid Y_i = 0)]^2\} \cdot p_i \\
&\quad + \{\operatorname{Var}(C_i \mid Y_i > 0) + [E(C_i \mid Y_i > 0)]^2\} \cdot (1 - p_i) \\
&= \{\operatorname{Var}(R_i) + \operatorname{Var}(M) + [E(R_i) + E(M)]^2\} \cdot p_i \\
&\quad + \{\operatorname{Var}(M) + [E(M)]^2\} \cdot (1 - p_i).
\end{aligned}$$

Substituting $\operatorname{Var}(R_i) = 1/a_i^2$, and simplifying yields the result.

5.6. *Numerical Example*

At this point it may be helpful to illustrate the various steps for obtaining state probabilities, means, and variances. As a specific example we use a gamma distribution for M with mean $E(M) = 800$ and standard deviation $SD(M) = 100$. Thus, the Laplace transform of M is

$$\rho(a) = [0.08/(0.08 + a)]^{64}. \tag{6}$$

Equation (6) is used for computing the transition probabilities via (4). We further choose $c = 3$, $n = 6$, and mean interarrival times of 700, 500, 800, 1000, 1500, and 3000 msec for $D_1, \ldots, D_6$, respectively.

The desired transition matrices $\mathbf{T}_1, \ldots, \mathbf{T}_6$ are then obtained by Definition (2) and Eq. (4):

$$\mathbf{T}_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.21 & 0.44 & 0.26 & 0.1 \\ 0.21 & 0.44 & 0.26 & 0.1 \\ 0 & 0.37 & 0.4 & 0.23 \\ 0 & 0 & 0.45 & 0.55 \end{bmatrix} \end{array}$$

$$\mathbf{T}_2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.37 & 0.4 & 0.18 & 0.04 \\ 0.37 & 0.4 & 0.18 & 0.04 \\ 0 & 0.45 & 0.41 & 0.14 \\ 0 & 0 & 0.59 & 0.41 \end{bmatrix} \end{array}$$

$$\mathbf{T}_3 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.45 & 0.41 & 0.13 & 0.01 \\ 0.45 & 0.41 & 0.13 & 0.01 \\ 0 & 0.59 & 0.36 & 0.06 \\ 0 & 0 & 0.77 & 0.23 \end{bmatrix} \end{array}$$

$$\mathbf{T}_4 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.59 & 0.36 & 0.06 & 0 \\ 0.59 & 0.36 & 0.06 & 0 \\ 0 & 0.77 & 0.23 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{T}_5 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.77 & 0.23 & 0 & 0 \\ 0.77 & 0.23 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{T}_6 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The reader may check the matrices with the aid of the transition diagram in Fig. 1 and with the sets $A_1, \cdots, A_6$ in Fig. 2.

The state probabilities are found by successively applying (5)

$$\begin{aligned} \mathbf{s}_1 &= (1.00, 0.00, 0.00, 0.00) \\ \mathbf{s}_2 &= (0.21, 0.44, 0.26, 0.10) \\ \mathbf{s}_3 &= (0.24, 0.38, 0.28, 0.10) \\ \mathbf{s}_4 &= (0.28, 0.42, 0.26, 0.05) \\ \mathbf{s}_5 &= (0.41, 0.44, 0.15, 0.00) \\ \mathbf{s}_6 &= (0.65, 0.35, 0.00, 0.00). \end{aligned}$$

For example, the probability that no copy waits in STS after completing the fourth recall is $P(Y_5=0)=0.41$.

The state probabilities $P(Y_i=0)$ $(i=1,..., 6)$ are then used in conjuction with Propositions (5) and (6) for computing the means and standard deviations of the intercompletion times C_i $(i=1,..., 6)$. They are shown in Table 1.

6. The Speed of Recall and STS-Capacity

In this section we point out some properties of the Simultaneous-Processing Model. Especially, we attempt to answer the following questions: First, how is STS capacity related to the speed of free recall? Second, do intercompletion times qualitatively reflect interarrival times?

TABLE 1

Mean Intercompletion Times (in msec) and Standard Deviations (in msec) as a Function of Recall Number i

	Recall number i					
	1	2	3	4	5	6
$E(C_i)$	1500	903	991	1079	1414	2764
$SD(C_i)$	707	320	529	700	1215	2817

Let us turn toward the first question. It is obvious by inspecting Proposition (5) that the lower bound for mean intercompletion time is given by mean motor-service time $E(M)$. Whether or not this lower bound may be approached depends on the mean interarrival time $E(D_i) = 1/a_i$ and on the probability $P(Y_i = 0)$. Among other things $P(Y_i = 0)$ depends reciprocally on STS capacity c. As c increases, $P(Y_i = 0)$ decreases and, therefore, the intercompletion time speeds up. A numerical example may be useful to point out this property more clearly. In Table 2 we computed mean intercompletion times for different values of c ($c = 0, 1, 2, 3, 4$) while keeping the other parameters constant ($n = 10$, $M \sim$Gamma with $E(M) = 800$ and $SD(M) = 100$). Mean interarrival times are computed according to the Random-Search Model($g = 0.05$ with mean sampling time $1/s = 30$), and shown in the bottom row of Table 2. One can observe that mean intercompletion time decreases as STS capacity increases. Already for $c = 1$ much time is saved if one compares the intercompletion times with those of $c = 0$ (no possibility to store copies in STS). However, there is no remarkable improvement with respect to time saved with larger values of c. These two features were revealed for many numerical examples.

TABLE 2

Mean Intercompletion Times for Different Values of c

	Recall number i									
STS-capacity c	1	2	3	4	5	6	7	8	9	10
0	1400	1467	1550	1657	1800	2000	2300	2800	3800	6800
1	1400	1003	1060	1139	1252	1418	1682	2142	3099	6052
2	1400	1003	979	1014	1099	1249	1505	1969	2943	5930
3	1400	1003	979	1001	1064	1188	1423	1877	2859	5872
4	1400	1003	979	1001	1062	1180	1403	1844	2818	5837
5	1400	1003	979	1001	1062	1180	1402	1839	2808	5924
6	1400	1003	979	1001	1062	1180	1402	1838	2806	5921
Interarrival times	600	667	750	857	1000	1200	1500	2000	3000	6000

Note. All values are rounded to the nearest millisecond.

A second feature in Table 2 may be noted. For two different capacities c_a and c_b ($c_a > c_b$), we notice that as long as $i < c_b$, corresponding mean intercompletion times do not differ for c_a and c_b. Only if $i > c_b + 1$ are mean intercompletion times smaller for c_a than for c_b.

Third, if we compare corresponding interarrival and intercompletion times a strange phenomenon occurs which may contradict one's intuition: Although, mean interarrival times increase with recall number i this is not necessarily true for mean intercompletion times. For example, consider the mean intercompletion times for $c > 1$. The first three intercompletion times actually decrease. This shows that mean intercompletion times need not even reflect in quality the temporal behavior of the LTS as long as one has to proceed from a STS buffer between motor channel and LTS. Therefore, it may be misleading to infer directly from intercompletion times the temporal behavior of the LTS—for example, one may be tempted to "conclude" that such a buffer only prolongs the intercompletion times by a constant motor time.

7. Delayed Free Recall: Evidence for the Simultaneous-Processing Model

In this section, we are concerned with an experiment addressed to the question of whether STS serves as a memory buffer in free recall. In this experiment the subject has the opportunity to preload STS with copies from LTS *before* the actual signal to start recall appears. This possibility to preload STS is achieved by instructing the subject to recall as many nouns as possible beginning with a specified initial letter and to withhold overt recall until the appearance of the actual recall signal. We hoped that the subject would take advantage of the delay to immediately begin the retrieval process and so preload STS.

The longer the delay the greater the probability that at least one copy has arrived at STS before presenting the actual recall signal. In addition, the greater this probability the lesser the mean intercompletion time (cf. Proposition 5). Hence, we expected the intercompletion times to decrease with increasing delay, especially for the first intercompletion times.

7.1. *Method*

The experiment was addressed to assess whether subjects utilize the delay between presenting the initial letter and the actual recall signal to preload STS with retrieved copies from LTS. There were two delay conditions. In condition *long* the period was 10 sec whereas in condition *short* it was 1 sec. In the framework of the Simultaneous-Processing Model we expect the first intercompletion times to be faster in condition long.

Subjects. Thirty students at Tübingen University participated in the experiment. Each subject participated at only one session for which they received course credit.

Design and Procedure. All sessions were conducted while the subject was seated in a sound-attenuated chamber. In front of the subject there was a PET computer which controlled the presentation of the initial letter, the delay, and the presentation of the actual recall signal.

For each subject there were 20 free-recall trials. One such trial had the following sequence: After the subject started the trial by pressing any key on the keyboard the initial letter appeared on the screen of the computer. The subject had to withhold an overt recall until the initial letter was replaced by the recall signal (START TO RECALL) on the screen. The subject was instructed to recall as many nouns as possible with the constraint that the recalled nouns must begin with the specified initial letter which was shown on the screen. After the subject had recalled 10 nouns the experimenter terminated the trial. After a pause of 30 sec the next trial began. A tape recorder was used to record the pronounced recalls. The intercompletion times were measured by using the tape-recorded recalls.

The initial letters were randomly drawn without replacement from the following 20 capital letters: A, B, C, D, E, F, G, H, I, J, K, M, N, O, P, R, S, T, U, V. Ten of the twenty trials were randomly assigned to delay condition short and the rest to condition long. The two delay conditions were presented in random order.

There were three trials at the beginning of the experiment using the initial letters C, W, Z to introduce the experiment. The session length was about 35 min.

7.2. *Results*

Mean intercompletion times for both delay conditions are shown in Fig. 4. As expected, mean intercompletion times are shorter at the beginning of the recall process for condition long than for condition short. This expected difference is also strengthened by a significant (delay condition) × (recall number) – interaction, $F(9,261) = 2.5, p < .001$. A comparison of mean intercompletion times for conditions long and short revealed a significant difference ($\alpha = .05$) up to recall number $i = 4$.

An unexpected and interesting result is the first intercompletion time in both delay conditions. The astonishing aspect, hereby, is that these times are much shorter than the second ones. Note that the first intercompletion time represents the time from onset of the actual recall signal up to completion of the first recall. Proceeding from the assumption that only STS (and not motor channel) is preloaded with copies from LTS, one would not expect such a difference between the means of C_1 and C_2. If at least two copies are waiting in STS before the actual recall signal appears then C_1 and C_2 just reflect the motor service times for the first and second copy. Thus, the means of C_1 and C_2 should be about equal (C_1 should even be larger than C_2 since one must take into account the time to perceive the actual recall signal). Therefore, the results suggest that the first retrieved copy immediately enters the motor channel and is prepared for the forthcoming recall before the actual recall signal appears. This conclusion would be consistent with the literature on response preparation. For example, Rosenbaum (1980) showed that choice reaction time was shorter if parts of the motor program can be prepared before presenting the actual response signal.

7.3. Predicting the Results with the Simultaneous-Processing Model

Let t denote the time delay, that is, the period beginning with the appearance of the initial letter and ending with the presentation of the actual recall signal. We assume that the subject preloads STS during the delay t with copies from LTS. We introduce the random variable $Y(t)$ to describe the number of copies occupying STS and motor channel at the very moment the actual recall signal is appearing.

To compute mean intercompletion times for predicting the results we only need to derive the probabilities $P[Y(t)=u]$, $(u=0, 1,..., c+1)$, that is, the probability that u copies queue in STS and motor channel when presenting the actual recall signal. Hence, the process need not start at coordinate $(0, n)$ when presenting the actual response signal. Any one of the following coordinate $(n-u, u)$, $u=0, 1,..., c+1$, is possible. Note that we assume a maximal preload of $c+1$ copies. This assumption is suggested by the hypothesis that one copy can also wait in motor channel where it is prepared for its forthcoming overt production.

The probabilities $P[Y(t)=u]$ have to be inserted into the initial vector $\mathbf{s}_1$ of (5). All other computation steps remain unchanged. The probability $P[Y(t)=u]$ is obtained by noting the relation between $Y(t)$ and the random variable $S_u=D_1+\cdots+D_u$, the time up to the uth arrival at STS. For it is clear from the definitions of $Y(t)$ and S_u that $Y(t)<u$ if and only if $S_u>t$.

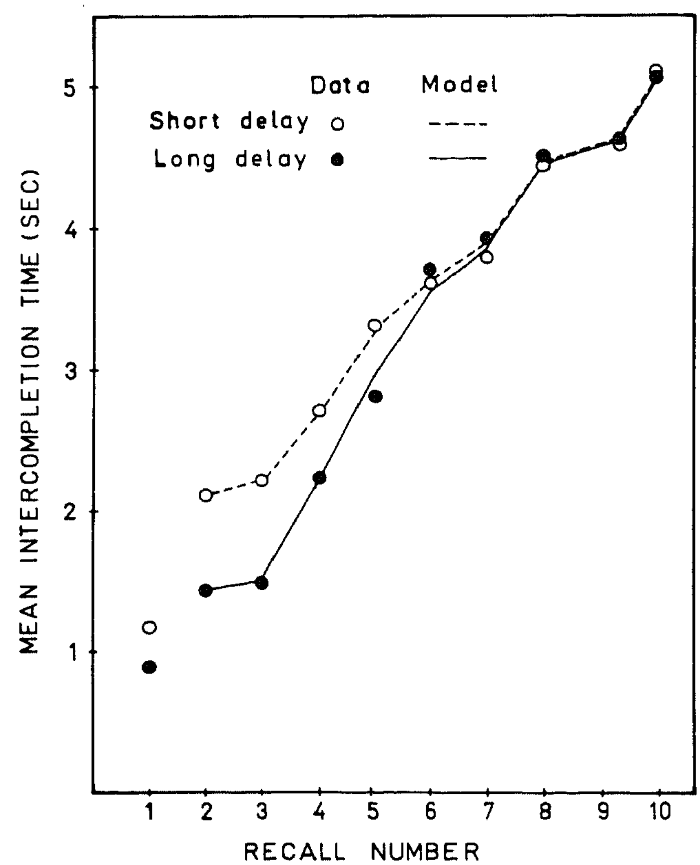

FIG. 4. Results of the delayed recall paradigm along the predictions of the Simultaneous-Processing Model—mean intercompletion times as a function of recall number for both delay conditions.

Thus,

$$P[Y(t) < u] = P[S_u > t]$$
$$= 1 - F_{S_u}(t),$$

where $F_{S_u}(t)$ is the distribution of S_u. Since the interarrival times are assumed to be exponentially distributed S_u follows the general Erlangian distribution given by (3). Hence, the probability that exactly u copies are prestored in STS and motor channel at time t is

$$P[Y(t) = u] = F_{S_u}(t) - F_{S_{u+1}}(t)$$

with (7)

$$F_{S_0}(t) = 1 \qquad \text{and} \qquad F_{S_{c+2}}(t) = 0.$$

We use (7) along with our previous derivations of the Simultaneous-Processing Model to predict the results of our experiment and to estimate the capacity of the STS. We therefore proceed as follows. Take any mean intercompletion time in Fig. 4—for example, the observed mean intercompletion time in delay condition short $\bar{C}_{5,1} = 3.32$ sec (the corresponding value in delay condition long is $\bar{C}_{5,2} = 2.83$ sec). The number of observations for each mean is $N = 300$. Let us write $E(C_{i,j} \mid \theta)$ and $SD(C_{i,j} \mid \theta)$ for the theoretical mean and standard deviation of intercompletion time $C_{i,j}$, respectively. The symbol θ denotes a vector and represents the model parameters, $\theta = (a_1, ..., c)$. If the model is true, and the parameter values in θ are accurate, the statistic

$$\frac{\bar{C}_{5,1} - E(C_{5,1} \mid \theta)}{SD(C_{5,1} \mid \theta)/\sqrt{299}}$$

is approximately normally distributed, with mean 0, and variance 1.

Consider now the expression

$$\chi^2(\theta) = \sum_{i=2}^{10} \sum_{j=1}^{2} \frac{(\bar{C}_{i,j} - E(C_{i,j} \mid \theta)]^2}{SD(C_{i,j} \mid \theta)^2/(N-1)}, \tag{8}$$

where the summation extends over all mean intercompletion times (with the omission of the first one in both delay conditions). Under the above assumption, $\chi^2(\hat{\theta})$ is approximately χ^2-distributed, with 5 degrees of freedom (18 observed means minus 13 parameters described below).

To estimate the parameters, we minimized (8) with the aid of the hill-climbing procedure (NAG Library, Subroutine E04JAF, 1978). We had to estimate 13 parameters: The 10 rates of the interarrival times, two parameters for the general-gamma distribution assumed for M, and one parameter for the STS capacity c. Minimization was done for each capacity separately up to $c = 10$. At each value of c we carried out 10 computer runs, always using different initial estimates for the 12 remaining parameters. A minimum was found at $c = 2$ with a value of $\chi^2(\hat{\theta}) = 6.8$.

The result is insignificant $.20 < p < 30$, supporting the Simultaneous-Processing Model. The fit is shown in Fig. 4.

It should be stressed that the parameter space is quite shallow, so that many other combinations of parameters give a fit of about the same goodness. This is especially true for the rates of the interarrival times and the parameters of the motor-service time distribution. However, a change of the capacity parameter c away from the optimal value always had a profound negative effect on the model fit. Because of this, c seems to be easier to identify than the remaining model parameters.

A capacity estimate of STS of $c = 2$ is often reported in the literature. Glanzer & Razel (1974, p. 119) surveyed 32 independent studies and reported a mean estimate of 2.2 words, with standard deviation of 0.64 words. Although our approach here is totally different from previous methods of measuring STS capacity we obtained about the same estimate and it is interesting to note that such a small capacity does predict a clear difference between the means of the two delay conditions up to the fifth recall.

8. Conclusion

The basic stochastic latency mechanism studied in cognitive psychology is one that assumes a series of stages or processes, such that the completion of a process immediately initiates the next process (Donders, 1868; Sternberg, 1969). Such a latency mechanism is appropriate for many tasks. However, if there is a continual arrival of new inputs from one stage to a further one—as in free recall or in copy typing—some principles must be added to the serial latency model. The Simultaneous-Processing Model may be viewed as a generalization of the serial latency model in this direction. Although, we have outlined the Simultaneous-Processing Model within the free-recall task, its framework may be applied and generalized further to a variety of tasks in which a continual arrival of new input from one stage to another must be assumed, e.g., typing or reading (Shaffer, 1973).

It should also be stressed that the model addresses important cognitive mechanisms in an analytic fashion, rather than by computer simulation. While no doubt models of great compexity must be subjected to simulation, we suspect that much more could be done with regard to analytic modeling of cognitive processes than is currently seen in the literature.

APPENDIX: The pdf of the Residual-Interarrival Time R

Proposition. *Suppose a random variable X with pdf $f_X(t)$ and an exponentially distributed variable D with $f_D(t) = a \cdot \exp(-at)$. Assume that X and D are started at $t = 0$ as it is shown in Fig. 3. Now let R denote the time elapsing between the com-*

pletions of X and D, that is, $R = D - X$. *We are concerned with trials for which* $R > 0$, *and state that* $f_R(t \mid R > 0) = a \cdot \exp(-at)$.[2]

Proof. First note that the cdf of R is

$$F_R(t \mid R > 0) = \frac{P(0 < R \leqslant t)}{P(R > 0)}.$$

The denominator is just the probability that X completes before D

$$\begin{aligned} P(R > 0) &= P(D > X) \\ &= 1 - \int_0^\infty f_X(t) \cdot F_D(t)\, dt \\ &= \int_0^\infty f_X(t) \cdot \exp(-at)\, dt. \end{aligned}$$

The numerator is the probability that X completes first and that the intercompletion time between X and D is less than t

$$\begin{aligned} P(0 < R \leqslant t) &= P(0 < D - X \leqslant t) \\ &= \int_0^\infty \int_x^{t+x} f_X(x) \cdot f_D(y)\, dy\, dx \\ &= \int_0^\infty f_X(x) \cdot (F_D(t+x) - F_D(x))\, dx \\ &= [1 - \exp(-at)] \cdot \int_0^\infty f_X(x) \cdot \exp(-ax)\, dx. \end{aligned}$$

Inserting the derived expressions for the numerator and denominator yields the desired result.

NOTE

Consider the case that STS is totally occupied with copies. One then might well ask what are the implications for the retrieval process: Is the retrieval process interrupted or not? A brief discussion of the two possibilities is in order.

[2] The proposition given here is of general interest. The memoryless property of the exponential distribution says that if one selects a *fixed* time point X before completion of the exponential process, then the time from X until completion has the same distribution as the total completion time (cf. Townsend & Ashby, 1983, p. 38). The above proposition shows that it does not matter whether X is *random or fixed,* the memoryless property holds in either case. There has been some confusion about this point in the recent literature (Fisher & Goldstein, 1983, p. 143).

Let us begin with the first possibility: The LTS process is interrupted as soon as the STS is filled up and restarted after the motor channel is freed. If restarting costs additional time then the arrival of the next copy is more delayed compared to the situation when STS was not filled up. The consequence is that the forthcoming residual-interarrival time may depend on whether the STS is filled up (then it is longer) or if there is at least one slot free in STS. The present version of the Simultaneous-Processing Model does not account for this possibility since it is implicitly assumed that the pdf of the residual-interarrival time does not depend on whether STS was filled up or not. If the probability that the STS will be filled up or if the additional time to restart the retrieval process is small, then the present version of the Simultaneous-Processing Model provides a good approximation.

However, this first possibility may not be so plausible as it seems at first, since absolute concurrent processing of LTS and motor channel is hindered by the occurrence of interruptions. Consequently, interruptions might delay the recall process unnecessarily.

The second possibility implies an absolute concurrent processing without interruptions and is consistent with the present version of the Simultaneous-Processing Model: The retrieval process is assumed to be going on irrespective of whether STS is filled. So one may ask what happens if a relevant item is retrieved and STS is filled up? Consider first the Random-Search Model: If a relevant item is retrieved while STS is filled the retrieved item will be replaced in the search area without being marked. At some time or other it will be resampled and transferred to STS. It follows that the forthcoming residual-interarrival time does not depend on whether STS was filled.

For the Parallel-Activation Model one may argue in an analogous manner: If an item is sufficiently activated while the STS is totally occupied then its copy cannot be transferred to STS. In this situation its activation will be immediately cancelled and a new activation cycle is started for it. At some time or other it will be activated while there is a free slot in STS and it can be transferred to STS. If all activation cycles have a common exponential distribution then the forthcoming residual-interarrival time does not depend on whether STS was filled.

References

Albert, D. (1968). Freies Reproduzieren von Wortreihen als stochastische Entleerung eines Speichers. *Zeitschrift für Experimentelle und Angewandte Psychologie*, **15**, 564–581.

Albert, D. (1972). *Zur Theorie der retroaktiven Hemmung*. Unpublished doctoral dissertation. Universität Marburg.

Ashby, F. G. (1982). Testing the assumptions of exponential, additive reaction time models. *Memory and Cognition*, **10**, 125–134.

Ashby, F.G., & Townsend, J. T. (1980). Decomposing the reaction time distribution: Pure insertion and selective influence revisited. *Journal of Mathematical Psychology*, **21**, 93–123.

Atkinson, R. C., & Shiffrin, R. M. (1968). Human memory: A proposed system and its control processes. In K. W. Spence & J. T. Spence (Eds.), *The psychology of learning and motivation: Advances in research and theory* (Vol. 2). New York: Academic Press.

Atkinson, R. C., & Shiffrin, R. M. (1971). The control of short-term memory. *Scientific American*, **224**, 82–90.

Bousfield, W. A., & Sedgewick, C. H. W. (1944). An analysis of sequences of restricted associative responses. *Journal of General Psychology*, **30**, 149–165.

Donders, F. C. (1868). Die Schnelligkeit Psychischer Prozesse. *Archiv für Anatomie und Physiologie und Wissenschaftliche Medizin*, 657–681.

Fisher, D. L., & Goldstein, W. M. (1983). Stochastic PERT networks as models of cognition: Derivation of the mean, variance, distribution of reaction time using order-of-processing (Op) diagrams. *Journal of Mathematical Psychology*, **27**, 121–151.

Glanzer, M., & Razel, M. (1974). The size of the unit in short-term storage. *Journal of Verbal Learning and Verbal Behavior*, **13**, 114–131.

Hinton, G. E., & Anderson, J. A. (Eds.) (1981). *Parallel models of associative memory*. Hillsdale, NJ: Erlbaum.

Kaplan, I. T., & Carvellas T. (1969). Response probabilities in verbal recall. *Journal of Verbal Learning and Verbal Behavior*, **8**, 344–349.

Kohfeld, D. L., Santee. J. L., & Wallace, N. D. (1981). Loudness and reaction time: II. Identification of detection components at different intensities and frequencies. *Perception & Psychophysics*, **29**, 550–562.

McGill, W. J. (1963). Stochastic latency mechanisms. In R. D. Luce, R. B. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (Vol. 1), New York: Wiley.

McGill, W. J., & Gibbon, J. (1965). The general-gamma distribution and reaction times. *Journal of Mathematical Psychology*, **2**, 1–18.

Murphy, M. D., & Puff, R. (1982). Free recall: Basic methodology and analyses. In C. R. Puff (Ed.), *Handbook of research methods in human memory and cognition*. New York: Academic Press.

NAG Library (1978). *Numerical algorithms group*, Subroutine E04ABF.

Raaijmakers, J. G. W., & Shiffrin, R. M. (1980). SAM: A theory of probabilistic search of associative memory. In G. Bower (Ed.), *The psychology of learning and motivation* (Vol. 14). New York: Academic Press.

Raaijmakers, J. G. W., & Shiffrin, R. M. (1981). Search of associative memory. *Psychological Review*, **88**, 93–134.

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review*, **85**, 59–108.

Rosenbaum, D. A. (1980). Human movement initiation: Specification of arm, direction, and extent. *Journal of Experimental Psychology: General*, **109**, 444–474.

Schulz, U., & Albert, D. (1976). Das Reproduzieren und sein Beendigung: Eine Klasse von Zählermodellen. *Zeitschrift für Experimentelle und Angewandte Psychologie*, **23**, 678–699.

Shaffer, L. H. (1973). Latency mechanisms in transcription. In S. Kornblum (Ed.), *Attention and performance* (Vol. 4). New York: Academic Press.

Shiffrin, R. M. (1975). Short-term store: The basic for a memory system. In F. Restle, R. M. Shiffrin, N. J. Castellan, H. Lindman, & D. B. Pisoni (Eds.), *Cognitive theory* (Vol. 1). Hillsdale, NJ: Erlbaum.

Shiffrin, R. M. (1976). Capacity limitations in information processing, attention and memory. In W. K. Estes (Ed.), *Handbook of learning and cognitive processes:* (Vol. 4), *Memory processes*. (pp. 177–236). Hillsdale, NJ; Erlbaum.

Sternberg, S. (1969). The discovery of processing stages: Extensions of Donders' method. In W. G. Koster (Ed.), *Attention and performance* (Vol. 2). Amsterdam: North-Holland.

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes*. Cambridge: Cambridge Univ. Press.

Vorberg, D., & Ulrich, R. (1985). *Random search with unequal search rates: Serial and parallel generalizations of McGill's model*. In preparation.

Waugh, N. C., & Norman, D. A. (1965). Primary memory. *Psychological Review*, **72**, 89–104.

Received: September 24, 1984